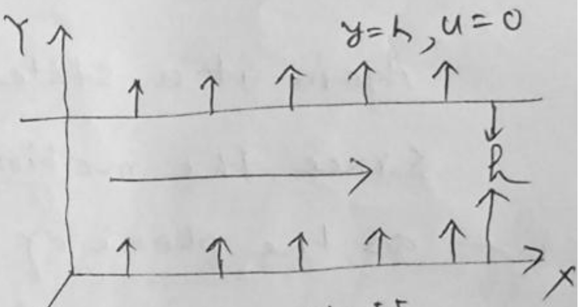
Ex. Incompressible viscous liquid is moving steadily under pressure between the planes $y=0$, $y=h$ the plane $y=0$ has a constant velocity U in the direction of x-axis, and the plane $y=h$ is fixed. The planes are porous and the liquid is sucked in uniformly over one and ejected over the other. Show that a possible solution is given by

$$u = \frac{U e^{h/2} + Ah - (U+Ah)\, e^{y/a}}{e^{h/a} - 1} + Ay.$$

$\vartheta = \frac{\nu}{a}$, where ν is the kinemate viscosity A and a are two constants.



Sol^n. Take x-axis along the lower plate in the direction of the flow, Y-axis normal to the direction of the main flow and z-axis along the weidth of the lower plate. It is clear that the motion in two-dimensional with the z-plane as the plane of the motion,

Let $\vec{q} = u\hat{i} + v\hat{j}$, be the flow velocity at the point $P(x,y,z)$ in the fluid.

The equation of continuity is

$$\vec{\nabla}\cdot\vec{q} = 0$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \longrightarrow (1)$$

Since the plate are infinite in length in x-direction

$\therefore$ u, v are independent of x.

$\therefore$ (1) $\Rightarrow \frac{\partial v}{\partial y} = 0$

$\therefore$ v is independent of y.

It is already stated that v is independent of x.

Since the motion is two dimensional so v is independent of z.

Further the motion is steady v is independent of t.

$\therefore$ v = a constant $= \frac{\nu}{a}$ (say)

Again it is stated that u is independent of x.

Since the motion is two-dimensional with the z-plane as the plane of the motion.

$\therefore$ u is independent of z.

Further u is independent of t as the motion is steady

$\therefore$ We must have $u = u(y) \longrightarrow$ (3)

The equation of motion for steady in absence of body force is

$$(\vec{q}\cdot\vec{\nabla})\vec{q} = -\frac{1}{\rho}\vec{\nabla}p + \nu\nabla^2\vec{q}$$

$$\Rightarrow \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)(u\hat{i} + v\hat{j}) = -\frac{1}{\rho}\vec{\nabla}p + \nu\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(u\hat{i} + v\hat{j})$$

$$\Rightarrow \frac{\nu}{a}\frac{d}{dy}\left[u\hat{i}+v\hat{j}\right] = -\frac{1}{\rho}\vec{\nabla}p + \nu\frac{d^2}{dy^2}\left(u\hat{i}+\frac{\nu}{a}\hat{j}\right)$$

$$\Rightarrow \frac{\nu}{a}\left[\frac{du}{dy}\hat{i}\right] = -\frac{1}{\rho}\vec{\nabla}p + \nu\frac{d^2u}{dy^2}\hat{i}$$

$$= -\frac{1}{\rho}\left(\hat{i}\frac{\partial p}{\partial x}+\hat{j}\frac{\partial p}{\partial y}+\hat{k}\frac{\partial p}{\partial z}\right) + \nu\frac{d^2u}{dy^2}\hat{i} \longrightarrow (4)$$

By equating the coefficient of $\hat{i}, \hat{j}, \hat{k}$ in (4), we derive the following differential equation

$$\frac{\nu}{a}\frac{du}{dy} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{d^2u}{dy^2} \longrightarrow (5.1)$$

$$\frac{\partial p}{\partial y} = 0 \longrightarrow (5.2)$$

$$\frac{\partial p}{\partial z} = 0 \longrightarrow (5.3).$$

(5.2) & (5.3) indicate that p is independent of y and z, also p is independent of t due to steady motion

(5.1) can be written as

$$\nu\frac{d^2u}{dy^2} - \frac{\nu}{a}\frac{du}{dy} = \frac{1}{\rho}\frac{dp}{dx}$$

$$\Rightarrow \mu\left[\frac{d^2u}{dy^2} - \frac{1}{a}\frac{du}{dy}\right) = \frac{dp}{dx} \longrightarrow (6)$$

$\Rightarrow$ a function of y = a function of x.

$$\therefore (6) \Rightarrow \mu\left[\frac{d^2u}{dy^2} - \frac{1}{a}\frac{du}{dy}\right] = \frac{dp}{dx} = \text{a constant} = -P \text{ (say)}$$

$$\Rightarrow \frac{d^2u}{dy^2} - \frac{1}{a}\frac{du}{dy} = -\frac{P}{\mu}$$

$$\Rightarrow \left[D^2 - \frac{1}{a} D\right] u = -\frac{P}{\mu} \longrightarrow (7)$$

A.E. is $m^2 - \frac{1}{a} m = 0$

$$\Rightarrow m\left(m - \frac{1}{a}\right) = 0$$

$$\Rightarrow m = 0, \frac{1}{a}$$

$$\therefore C.F. = c_1 + c_2 e^{\frac{y}{a}}$$

& $P.I. = -\dfrac{P/\mu}{D\left(D - \frac{1}{a}\right)}$

$$= -\frac{P}{\mu} \cdot \frac{1}{D} \frac{1}{D - \frac{1}{a}}$$

$$= -\frac{P}{\mu} \cdot \frac{1}{D} \cdot \frac{1}{-\frac{1}{a}}$$

$$= \frac{P}{\mu} \frac{1}{D}(a)$$

$$= \frac{aP}{\mu} y$$

$$= Ay \quad , \quad A = \frac{Pa}{\mu}$$

$$\therefore u = c_1 + c_2 e^{y/a} + Ay \longrightarrow (8)$$

The boundary condition for the present flow problem are

$$u = U \text{ at } y = 0 \longrightarrow (9.1)$$

$$u = 0 \text{ at } y = h \longrightarrow (9.2)$$

Subjecting (8) to (9.1) and (9.2) we get,

$$U = c_1 + c_2 \longrightarrow (10.1)$$

$$0 = c_1 + c_2 e^{y/a} + Ah \longrightarrow (10.2)$$

$$\therefore (10.1) - (10.2) \Rightarrow U = c_2\left(1 - e^{h/a}\right) - Ah$$

$$\Rightarrow c_2 = \frac{U + Ah}{1 - e^{h/a}}$$

$$\therefore (8) - (10.1) \Rightarrow u - U = c_2\left(e^{\frac{y}{a}} - 1\right) + Ay$$

$$\Rightarrow u = U + \frac{(U + Ah)\left(e^{\frac{y}{a}} - 1\right)}{1 - e^{y/a}} + Ay$$

$$= U + \frac{U + Ah}{e^{h/a} - 1}\left(1 - e^{y/a}\right) + Ay$$

$$= \frac{U\left(e^{h/a} - 1\right) + (U + Ah)\left(1 - e^{\frac{y}{a}}\right)}{\left(e^{h/a} - 1\right)} + Ay$$

$$= \frac{Ue^{h/a} - U + U - Ue^{\frac{y}{a}} + Ah - Ahe^{\frac{y}{a}}}{\left(e^{h/a} - 1\right)} + Ay$$

$$= \frac{Ue^{h/a} + Ah - (U + Ah)e^{\frac{y}{a}}}{\left(e^{h/a} - 1\right)} + Ay$$

$\therefore$ The solution of the flow problem are

$$u = \frac{Ue^{h/a} + Ah - (U + Ah)e^{y/a}}{\left(e^{h/a} - 1\right)} + Ay.$$

and $v = \frac{\nu}{a}$

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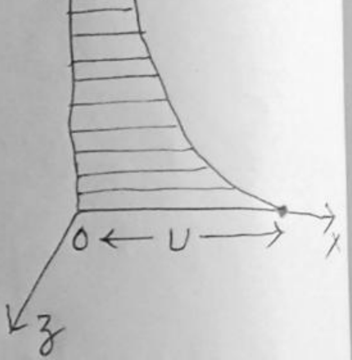
Ex. 14. The space above ~~problem~~ the plane $y=0$ is filled with a liquid of kinematic viscosity ν. Initially the plane and the liquid are at rest. At time $t=0$, the plane suddenly being to move parallel to itself with a constant velocity U and the liquid also moves in the same direction. Show that the solution of the flow problem is given by

$$u = U\left[1 - \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta\right], \text{ where } \eta = \frac{y}{2\sqrt{\nu t}}.$$

Soln. Take x-axis along the direction of the flow and y-axis perpendicular to the flow.



Here the motion in 2-D with the z-plane as the plane of the motion.

Let $\vec{q} = (u, v, w)$ be the fluid velocity at a point $P(x, y, z, t)$ in the flow.

In the present case, since the motion in 2-D with the z-plane as the plane of the motion as mentioned earlier,

$$\therefore\ w = 0$$

further, as the flow is parallel to x-axis

$$\therefore\ v = 0$$

Hence, $\vec{q} = u\hat{i}$, where $\hat{i}$ is the unit vector along $\vec{OX}$.

The equation of continuity for incompressible fluid is

$$\vec{\nabla}\cdot\vec{q} = 0$$

$$\Rightarrow \frac{\partial u}{\partial x} = 0$$

$\therefore$ u is independent of x.

as the motion is 2-D with the z-plane as the plane of the motion.

$\therefore$ u is independent of z

$$\therefore u = u(y,t) \longrightarrow (1)$$

The equation of motion for an incompressible viscous fluid with constant pressure is absence of body force is

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\vec{\nabla})\vec{q} = \nu\nabla^2\vec{q} \longrightarrow (2)$$

We have, $\vec{q} = u\hat{i}$

$$\therefore \vec{q}\cdot\vec{\nabla} = u\frac{\partial}{\partial x}$$

$$\therefore (\vec{q}\cdot\vec{\nabla})\vec{q} = \left(u\frac{\partial}{\partial x}\right)u\hat{i}$$

$$= u\frac{\partial u}{\partial x}\hat{i}$$

$$= 0. \quad \because u = u(y,t)$$

$\therefore$ equation (2) becomes to

$$\frac{\partial \vec{q}}{\partial t} = \nu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\vec{q}$$

$$\Rightarrow \frac{\partial}{\partial t}(u\hat{i}) = \nu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)u\hat{i}$$

$$\Rightarrow \frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \longrightarrow (3)$$

Now the solve the equation (3) subject to the initial and boundary conditions

$$u = 0 \quad \text{when} \quad t \leq 0 \ \forall y \longrightarrow (4.1)$$

$$u = \begin{cases} U & \text{at } y = 0 \\ 0 & \text{at } y \to \infty \end{cases} \quad \forall t > 0 \longrightarrow (4.2)$$

Let us consider the transformation

$$u = U f(\eta) \longrightarrow (5)$$

$$\text{where } \eta = \frac{y}{2\sqrt{\nu t}} \longrightarrow (6)$$

Now, $\frac{\partial u}{\partial t} = U f'(\eta) \frac{\partial \eta}{\partial t}$

$$= U f'(\eta) \frac{\partial \eta}{\partial t} \frac{y}{2\sqrt{\nu}} t^{-\frac{1}{2}}$$

$$= \frac{U y f'(\eta) \left(-\frac{1}{2}\right) t^{-3/2}}{2\sqrt{\nu}}$$

$$= -\frac{U y f'(\eta)}{4\sqrt{\nu}\, t\sqrt{t}}$$

$$= -\frac{U \eta f'(\eta)}{2t}$$

$$\frac{\partial u}{\partial y} = U f'(\eta) \frac{\partial \eta}{\partial y} = U f'(\eta) \frac{\partial}{\partial y} \frac{y}{2\sqrt{\nu t}}$$

$$= \frac{U f'(\eta)}{2\sqrt{\nu t}}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{U}{2\sqrt{\nu t}} f''(\eta) \frac{\partial \eta}{\partial y}$$

$$= \frac{U}{2\sqrt{\nu t}} f''(\eta) \frac{1}{2\sqrt{\nu t}}$$

$$= \frac{U f''(\eta)}{4\nu t}$$

$$\therefore (3) \Rightarrow -\frac{U\eta f'(\eta)}{2t} = \nu \frac{U f''(\eta)}{4\nu t}$$

$$\Rightarrow -\eta f' = \frac{f''}{2}$$

$$\Rightarrow f'' + 2\eta f' = 0$$

$$\Rightarrow \frac{df'}{d\eta} + 2\eta f' = 0$$

$$\Rightarrow \frac{df'}{f'} + 2\eta \, d\eta = 0$$

$$\Rightarrow \log f' + \eta^2 = \log A$$

$$\Rightarrow \log \frac{f'}{A} = -\eta^2$$

$$\Rightarrow f' = A e^{-\eta^2}$$

$$\Rightarrow df = A e^{-\eta^2} d\eta$$

$$\Rightarrow f = A \int_0^{\eta} e^{-\eta^2} d\eta + B \longrightarrow (7)$$

Where A and B are two arbitrary constants to be determined from the initial and boundary conditions of the problem.

For the present problem: $u = U$ at $y = 0$, $t > 0$

$\therefore\ Uf'(\eta) = U\ ,\ \eta = 0$

i.e. $f(\eta) = 1$ at $\eta = 0$

i.e. $f(0) = 1$.

Again, $u = 0$ at $y \to \infty,\ t > 0$

i.e. $Uf(\eta) = 0$ at $\eta \to \infty$

$\Rightarrow f(\eta) = 0$ at $\eta \to \infty$

i.e. $f(\infty) = 0 \longrightarrow (8.2)$

Using (8.1) in (7), we have,

$1 = A \cdot 0 + B$

$\Rightarrow B = 1$

Using (8.2) in (7), we have,

$$0 = A\int_0^\infty e^{-\eta^2} d\eta + 1$$

$$\Rightarrow 0 = AI + 1 \longrightarrow (9)$$

Now, $I = \int_0^\infty e^{-\eta^2} d\eta$;

Let $\eta^2 = \xi$

$2\eta\, d\eta = d\xi$

$\Rightarrow d\eta = \frac{d\xi}{2\eta} = \frac{d\xi}{2\sqrt{\xi}}$

$\therefore \eta = 0,\ \xi = 0$

$\eta \to \infty,\ \xi \to \infty$

$$= \int_0^\infty e^{-\xi} \frac{d\xi}{2\sqrt{\xi}}$$

$$= \frac{1}{2}\int_0^\infty \xi^{-\frac{1}{2}} e^{-\xi} d\xi$$

$$= \frac{1}{2}\int_0^\xi \xi^{\frac{1}{2}-1} e^{-\xi} d\xi$$

$$= \frac{1}{2} \cdot \Gamma\tfrac{1}{2} \qquad , \because \int_0^\infty x^{\eta-1} e^{-x} dx = \Gamma\eta . \quad (11)$$

$$= \frac{\sqrt{\pi}}{2}$$

$\therefore$ (9) $\Rightarrow$ $0 = A \frac{\sqrt{\pi}}{2} + 1$

$\Rightarrow A = -\frac{2}{\sqrt{\pi}}$

$\therefore$ (7) $\Rightarrow$ $f(\eta) = -\frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\eta^2} d\eta + 1$

$$= 1 - \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\eta^2} d\eta$$

$\therefore$ The velocity for the flow problem is given by

$$u = U \left[1 - \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\eta^2} d\eta \right]$$

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Note: $u = U \left[1 - erfc(\eta) \right]$

Reynolds Number:

The Reynolds no. Re is defined as

$$Re = \frac{\text{Inertia term}}{\text{viscous force}} = \frac{\text{Inertia force}}{\text{viscous force}}$$

$$= \frac{u \frac{\partial u}{\partial x}}{\nu \frac{\partial^2 u}{\partial x^2}}$$

$$= \frac{U \cdot \frac{U}{L}}{\nu \frac{U}{L^2}}$$

$$= \frac{U^2}{L} \times \frac{L^2}{\nu U}$$

$$= \frac{UL}{\nu}$$, where U is the characteristic velocity, L is the characteristic length and ν is the kinematic viscosity.

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(113)

Ex. Show that for a steady axi-symmetric flow at small Reynolds no. Stokes stream function ψ satisfies the equation

$$E^4(\psi) = 0 \text{ where } E^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right).$$

Sol^n. Let $\vec{q} = q_r\hat{r} + q_\theta\hat{\theta} + q_\phi\hat{\phi}$ be the fluid velocity at a point $P(r,\theta,\phi)$ in the fluid.

Since the motion is symmetrical about the axis

$\therefore \theta = 0$

$\therefore q_\phi = 0$

and q_r, q_θ are independent of ϕ

The equation of motion for an incompressible viscous fluid is

$$\frac{\partial\vec{q}}{\partial t} + (\vec{q}\cdot\vec{\nabla})\vec{q} = \vec{F} - \frac{1}{\rho}\vec{\nabla}p - \nu\,\vec{\nabla}\times\vec{\xi} \longrightarrow (A)$$

In the present case,

1, The motion is steady for which $\frac{\partial\vec{q}}{\partial t} = 0$

2. There is no external force and hence $\vec{F} = 0$

3. R_e is small.

$\therefore (\vec{q}\cdot\vec{\nabla})\vec{q} \longrightarrow 0$ i.e. $(\vec{q}\cdot\vec{\nabla})\vec{q} = \vec{0}$

$\therefore$ The equation (A) reduces to

$$0 = -\frac{1}{\rho}\vec{\nabla}p - \nu\,\vec{\nabla}\times\vec{\xi}$$

$$\Rightarrow \vec{\nabla} p = -\mu \, \vec{\nabla} \times \vec{\xi}$$

$$\Rightarrow \vec{\nabla} \times \vec{\nabla} p = -\mu \, \vec{\nabla} \times (\vec{\nabla} \times \vec{\xi})$$

$$\Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{\xi}) = 0 \longrightarrow (1)$$

Since ψ is the stokes stream function

$$\left.\begin{aligned} \therefore q_r &= -\frac{1}{r^2 \sin\theta} \frac{\partial \psi}{\partial \theta} \\ q_\theta &= \frac{1}{r \sin\theta} \frac{\partial \psi}{\partial r} \end{aligned}\right\} \longrightarrow (2)$$

Now, $\vec{\xi} = \vec{\nabla} \times \vec{q}$

$$= \frac{1}{1 \cdot r \cdot r\sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta \, \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ q_r & r \cdot q_\theta & r\sin\theta \, q_\phi \end{vmatrix}$$

$$= \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta \, \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ q_r & r q_\theta & 0 \end{vmatrix}$$

$$= \frac{1}{r^2 \sin\theta} \cdot r\sin\theta \, \hat{\phi} \left\{ \frac{\partial}{\partial r}(r q_\theta) - \frac{\partial q_r}{\partial \theta} \right\}$$

$$= \frac{\hat{\phi}}{r} \left[\frac{\partial}{\partial r} \left\{ r \cdot \frac{1}{r\sin\theta} \frac{\partial \psi}{\partial r} \right\} + \frac{\partial}{\partial \theta} \left\{ \frac{1}{r^2} \frac{1}{\sin\theta} \frac{\partial \psi}{\partial \theta} \right\} \right]$$

$$= \frac{\hat{\phi}}{r} \left[\frac{\partial}{\partial r} \left[\frac{1}{\sin\theta} \frac{\partial \psi}{\partial r} \right] + \frac{\partial}{\partial \theta} \left(\frac{1}{r^2 \sin\theta} \frac{\partial \psi}{\partial \theta} \right) \right]$$

$$= \frac{\hat{\phi}}{r\sin\theta}\left[\frac{\partial^2\psi}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\psi}{\partial\theta}\right)\right]$$

$$= \frac{\hat{\phi}}{r\sin\theta}\left[\frac{\partial^2\psi}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)\right]\psi$$

$$= \frac{\hat{\phi}}{r\sin\theta} E^2(\psi)$$

$$= \frac{E\phi}{r\sin\theta}\hat{\phi}, \quad E\phi = E^2(\psi)$$

$$\vec{\nabla}\times\vec{\xi} = \frac{1}{1.r.r\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 0 & r\sin\theta\,\frac{E\phi}{r\sin\theta} \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 0 & E\phi \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\left[\hat{r}\frac{\partial E\phi}{\partial\theta} - r\hat{\theta}\frac{\partial E\phi}{\partial r}\right]$$

$$= \hat{r}\,\frac{1}{r^2\sin\theta}\frac{\partial E\phi}{\partial\theta} - \frac{\hat{\theta}}{r\sin\theta}\frac{\partial E\phi}{\partial r}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{\xi}) = \frac{1}{1.r.r\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ \frac{1}{r^2\sin\theta}\frac{\partial E\phi}{\partial\theta} & -r\frac{1}{r\sin\theta}\frac{\partial E\phi}{\partial r} & 0 \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta} \cdot r\sin\theta\, \hat{\phi} \left[-\frac{\partial}{\partial r}\left(\frac{1}{\sin\theta} \frac{\partial E\phi}{\partial r}\right) - \frac{\partial}{\partial\theta}\left(\frac{1}{r^2\sin\theta} \frac{\partial E\phi}{\partial\theta}\right) \right]$$ 116

$$= -\frac{\hat{\phi}}{r}\left[\frac{1}{\sin\theta}\frac{\partial^2 E\phi}{\partial r^2} + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial E\phi}{\partial\theta}\right)\right]$$

$$= -\frac{\hat{\phi}}{r\sin\theta}\left[\frac{\partial^2 E\phi}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial E\phi}{\partial\theta}\right)\right]$$

$$= -\frac{\hat{\phi}}{r\sin\theta}\left[\frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)\right]E\phi$$

$$= -\frac{\hat{\phi}}{r\sin\theta} E^2(E\phi) = -\frac{\hat{\phi}}{r\sin\theta} E^2(E^2(\psi))$$

$$= -\frac{\hat{\phi}}{r\sin\theta} E^4(\psi)$$

$\therefore$ The equation (1) gives

$$-\frac{\hat{\phi}}{r\sin\theta} E^4(\psi) = 0 \Rightarrow E^4(\psi) = 0 .$$ #