Game Theory

The theory of games was developed by Jhon Von Neumann and Oscar Morgenstern in their famous work edited "The Theory of Games and Economic behaviour" in 1944. This theory determines the standards of rational behaviour where the outcomes depend on the actions of indipendent individuals. In other words the aim of game Theory is to work out recommendations for the rational behaviour in conflicting situations – that is to determine an optimal strategy for each player. The game Theory is applicable to diverse economic problems such as situation of duopoly, oligopoly, bilateral monopoly. However, let us consider a duopoly game.

As we know a duopolist (a player of the game) has to select one among a variety of possible courses of action technically known as strategies.

Each duopolist is assumed to possess a finite number of strategies though the number may be very large. A dupolist strategy may consist of selecting a particular price. A price and advertising expenditure ~~expenditure~~ are both variables. A strategy consists of selecting particular values for both price and advertising expenditure. The outcome of the duopolistic game - that is the profit earned by each of the participants is determined from the relevant cost and demand relation once each of the duopolists has selected strategy.

Assumptions :

(1) The game theory assumes that while selecting strategy a duopolist will assume that his rival will adopt a strategy which will be most unfavourable to his interest.

(2) Game theory also assumes that a duopolist knows all the possible strategies open to him as well as those strategies open to his rival.

Games are classified on the basis of two criteria —

(1) The no. of participating firms or persons

(2) The net outcome.

The first criterion involves accounting of the no. of persons with conflicting interest. There are one person, two person, three person and in general n- person game.

The second criterion distinguishes between zero-sum and non zero sum. A zero-sum game is that in which the algebraic sum of the outcome, for example say, profits for all the participating firms, equals zero for every possible combination of strategies. If the net outcome of a game is different from zero for at least one strategy combination, it is than classified as non-zero sum game.

A one person zero-sum game is uninteresting since the player gains nothing regardless of the strategy that he chooses.

Two-person zero-sum game is applied to a duopolistic market in which one duopolists gains are always equal to other's losses. The two-person zero-sum game is also termed as two-person constant-sum game.

Here the theory is applied to duopolistic market in which a pair of duopolist. ~~market in which a pair of duopolist~~ is competing for some given total profit. What is gained by one is lost by the other.

In other words, the 'constant-sum' game implies that the profits to the two duopolists always adopt to a constant amount. For example, suppose the profits to be shared by the two duopolists amount to Rs 100. If duopolist I gets Rs 60, then duopolist II will get Rs 40. If on the other hand duopolist I gets Rs 30 then duopolist II will get Rs 70, so that the aggregate will always adds up to Rs 100..

Now, the duopolists may have their different strategies to adopt so that each of them may be able to achieve its objective. In general, if duopolist I has m strategies and duopolist II has n strategies then the possible outcomes of the game are given by the following profit matrix. The profit matrix is also known as pay-off matrix.

$$\begin{vmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{mn} \end{vmatrix}_{m\times n}$$

In general, x_{ij} $\left(\begin{matrix} i = 1, 2, \cdots m \\ j = 1, 2, \cdots n \end{matrix}\right)$ is the profit of duopolist I if duopolist I adopts i^{th} strategy and duopolist II adopts j^{th} strategy.

Since the game is zero-sum, the corresponding profit earned by duopolist II will be $(-x_{ij})$.

Neumann-Morgenstern Game Theory

In order to discuss the solution of oligopoly problem suggested by Neumann-Morgenstern's theory of game, we suppose (1) that an oligopoly knows the complete set of strategies open to him as well as those available to his rivals in the industry.

(2) while selecting a strategy an oligopolist will assume that his rival will adopt a strategy which will be most unfavourable to his interest.

(3) Lastly we take a constant-sum-game, in which the outcomes to the two players ie oligopolists always add up to the same constant amount. That is one player's gain is always another player's loss.

In our explanation of the game theory, we describe the behaviour of a pair of duopolist A and B who compete for a given total profits of Rs 10. when in a game aggregate of gain (+) and loss (-) is zero, the constant-sum game becomes a zero-sum-game. In the oligopolistic market situation, if advertising campaign launch-ed by a firm for promoting the sales of its

Product merely causes a fixed number of consumers to switch from other brands to his brand without adding to the total demand

Maximin and Minimax strategies

it is an example of zero sum game

Suppose three strategies A_1, A_2, A_3 are open to A and three strategies B_1, B_2, B_3 are open to B. The amount of profits which accrue to A as a result of the strategies adopted by him and his rival B is given by the following A's pay-off matrix.

A's strategies	B's strategies: B_1	B_2	B_3	Row minimum
A_1	2	8	1	1
A_2	4	3	9	3
A_3	5	6	7	5
Column maximum	5	8	9	

Thus, if A adopts strategy A_1 and B adopts B_1, then profit to A are 2. The profit of B will be $(10-2)=8$, since it is two persons-zero-sum game. Likewise when A adopts A_1, B adopts B_2 the profit of A is 8 and so on.

Now given the A's pay-off matrix, the
problem is which strategy will be selected by
A and B and what will be the outcome.
Suppose A chooses first. By assumption, A will
choose his strategy keeping in mind that B will
adopt the most unfavourable strategy for him.
Thus it is clear that when A adopts A_1, B
adopts B_3; when A adopts A_2, B adopts B_2 and
when A adopts A_3, B adopts B_1 so as to cause
minimum possible profit to A. Now out of
these three, A will choose A_3, so that if B
plays the most unfavourable strategy, even then he
gets profits of 5 which is greater than the
other two minimum profits of 1 and 3. Thus
A will choose maximum of the 'row minima'.
That is A will follow a <u>maximin strategy</u>.

When A has selected A_3 and B comes to know it, B will adopt B_1 so as to cause minimum possible profit to A (5) and ensure maximum possible profit to himself which is $(10-5)=5$

Now suppose B had to choose first. By same argument, if B chooses B_1, A chooses A_3, when B chooses B_2, A chooses A_1 and when B chooses B_3, A chooes A_2 so as to cause maximum possible profit to him and minimum possible profit to B. Now out of these three strategies, if B adopts B_1 then A's profit will be minimum (5). Thus B chooes minimum of the column maxima. Thus he plays a <u>minimax strategy</u>.

B has chosen B_1 and announce it, A will examine his strategies and choose A_3 so as to get maximum possible profit (5).

It should be noted that in the actual game in the real world, neither A nor B has to choose first, they choose simultaneously. But the same argument applies. Given A's pay off matrix, A will play <u>maximin strategy</u> and B will play <u>minimax strategy</u>.

<u>Equilibrium point or Saddle point</u>:

In the pay-off matrix of our analysis, A's maximin strategy coincides with B's minimax strategy. When it happens so, the pay-off matrix is said to possess an equilibrium point or, what is also technically called 'saddle point'. Thus a saddle point exists when

maximin strategy = minimax strategy.

When there is no saddle point in the pay-off matrix, then stable equilibrium will not be attained. In such cases, it matters a great deal who plays first and also it is helpful to know rival's strategy in advance. Various methods have been developed to provide solution in such a pay-off matrix. One of these methods is to permit the players to employ mixed strategies. A mixed strategy is a combination of two strategies with the probabilities assigned to these strategies.

(Note the technique involved is too complicated and not attempted here.)