Frobenius method of Series Solution (when $x=0$ is Regular Singular Point)

FLOW-CHART:-

Homogeneous second order linear Differential Eq. with variable coefficients (Assume $x=0$ is Regular Singular Point)

Assume the solution $y = \sum_{n=0}^{\infty} a_n x^{m+n}$ and find $\frac{dy}{dx}, \frac{d^2y}{dx^2} = ?$

m can have any value

Substitute the values of $y, \frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the given Diffi. Eq.

Imp. Put the coefficient of lowest power of x to zero, we will get an equation in terms of m called as "Indical Eq."

Find the values / roots of Indical Eq. On the basis of values of m, it is further categorized in 3 Sub cases:-

Case 1:- $m_1 \neq m_2$ and $m_1 - m_2 \neq$ Integer (m_1 & m_2 are distinct)

Case 2:- $m_1 = m_2$ (m_1 & m_2 are same)

Case 3:- $m_1 \neq m_2$ and $m_1 - m_2 =$ Integer (m_1 & m_2 are distinct)

Imp By Applying Indexing, Reduce all powers of x to x^{m+n}

Equate the Coefficient of x^{m+n} to zero, it will give a Recurrance Relation or difference Eq. and Find the values of $a_1, a_2, a_3 \ldots$

Substitute the values of $a_1, a_2, a_3, a_4 \ldots$ in assumed solution to get one of the solution. The complete solution will depends on the nature of values/roots of m (i.e on above 3 cases)

Case I:– When Roots m_1 & m_2 are distinct $(m_1 \neq m_2)$ & $m_1 - m_2 \neq$ Integer

Trial Solution :– $\boxed{y = \sum_{n=0}^{\infty} a_n x^{m+n}}$; Complete Sol. :– $\boxed{y = c_1 (y)_{m_1} + c_2 (y)_{m_2}}$

Q.5 :– Using Frobenius method of series, solve

Imp:–

$$2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0$$

about the Regular singular Point $x = 0$.

OR

Solve the Diff. Eq. $2x(1-x)y'' + (1-x)y' + 3y = 0$ in series

→ about the point $x=0$

→ near the point $x=0$

→ in the powers of x [UPTU, U.P.– 2018, 2016, 2004]

Sol:- According to the question :-

$$2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0 \quad \text{—— (1)}$$

Which is a Homogeneous Second order Linear Differential Equation with variable coefficients.

Here :- $P_0(x) = 2x(1-x)$; $P_1(x) = 1-x$; $P_2(x) = 3$

Now at Point $x=0$:- $P_0(0) = 2(0)(1-0) = 0 \rightarrow$ Singular Point

Now :-

$$\boxed{\lim_{x\to a} (x-a)\frac{P_1(x)}{P_0(x)}}$$

$$\lim_{x\to 0} \frac{(\cancel{x})\cancel{(1-x)}}{2\cancel{x}\cancel{(1-x)}} = \boxed{\frac{1}{2}}$$

Finite

Analytic

$$\boxed{\lim_{x\to a} (x-a)^2\frac{P_2(x)}{P_0(x)}}$$

$$\lim_{x\to 0} (x-0)^2 \frac{3}{2x(1-x)}$$

$$\lim_{x\to 0} \overset{x}{\cancel{x^2}} \frac{3}{2\cancel{x}(1-x)} = \boxed{0}$$

Finite

$\rightarrow$ Regular Singular Point

So $x = 0$ is a Regular Singular Point of given Diff Eq. (36)

Now by using Frobenius Method:-

m can have any value

Let $y = \sum_{n=0}^{\infty} a_n x^{m+n}$

$$\frac{dy}{dx} = \sum_{\boxed{n=0}}^{\infty} (m+n)\, a_n\, x^{m+n-1}$$ → Note

$$\frac{d^2y}{dx^2} = \sum_{\boxed{n=0}}^{\infty} (m+n)(m+n-1)\, a_n\, x^{m+n-2}$$ → Note

Now, put the values of y, $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ in given D.E (1), we get:-

$$2x(1-x)\left[\sum_{n=0}^{\infty}(m+n)(m+n-1)\, a_n\, x^{m+n-2}\right] + (1-x)\left[\sum_{n=0}^{\infty}(m+n)\, a_n\, x^{m+n-1}\right]$$
$$+\ 3\left(\sum_{n=0}^{\infty} a_n x^{m+n}\right) = 0$$

$$\Rightarrow (2x - 2x^2)\left[\sum_{n=0}^{\infty}(m+n)(m+n-1)\, a_n\, x^{m+n-2}\right] + (1-x)\left[\sum_{n=0}^{\infty}(m+n)\, a_n\, x^{m+n-1}\right]$$

$$+ 3\left[\sum_{n=0}^{\infty} a_n\, x^{m+n}\right] = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} 2(m+n)(m+n-1)\, a_n\, x^{m+n-1} - \sum_{n=0}^{\infty} 2(m+n)(m+n-1)\, a_n\, x^{m+n}$$

$$+ \sum_{n=0}^{\infty} (m+n)\, a_n\, x^{m+n-1} - \sum_{n=0}^{\infty} (m+n)\, a_n\, x^{m+n} + \sum_{n=0}^{\infty} 3 a_n\, x^{m+n} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty}\left[2(m+n)(m+n-1) + (m+n)\right] a_n\, x^{m+n-1}$$

$$- \sum_{n=0}^{\infty}\left[2(m+n)(m+n-1) + (m+n) - 3\right] a_n\, x^{m+n} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} \left[2(m+n-1)+1\right](m+n)\,a_n\,x^{m+n-1} - \sum_{n=0}^{\infty}\left[\left[2(m+n-1)+1\right](m+n)-3\right]a_n\,x^{m+n}=0$$

$$\Rightarrow \sum_{n=0}^{\infty} (2m+2n-1)(m+n)\,a_n\,x^{m+n-1} - \sum_{n=0}^{\infty}\left[(2m+2n-1)(m+n)-3\right]a_n\,x^{m+n}=0$$

$$\Rightarrow \sum_{n=0}^{\infty} (2m+2n-1)(m+n)\,a_n\,x^{m+n-1} - \sum_{n=0}^{\infty}\left[\left(2(m+n)-1\right)(m+n)-3\right]a_n\,x^{m+n}=0$$

$$\Rightarrow \sum_{n=0}^{\infty} (2m+2n-1)(m+n)\,a_n\,x^{m+n-1} - \sum_{n=0}^{\infty}\left[2\underline{(m+n)}^2-\underline{(m+n)}-3\right]a_n\,x^{m+n}=0$$

(39)

$$\Rightarrow \sum_{n=0}^{\infty} (2m+2n-1)(m+n)\, a_n\, x^{m+n-1} - \sum_{n=0}^{\infty} \left[\underline{2(m+n)^2 - 3(m+n)} + \underline{2(m+n) - 3} \right] a_n x^{m+n} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (2m+2n-1)(m+n)\, a_n\, x^{m+n-1} - \sum_{n=0}^{\infty} \left[(m+n)\left(\underline{2(m+n)-3}\right) + 1\left[\underline{2(m+n)-3}\right] \right] a_n x^{m+n} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (2m+2n-1)(m+n)\, a_n\, x^{m+n-1} - \sum_{n=0}^{\infty} (2m+2n-3)(m+n+1)\, a_n\, x^{m+n} = 0 \quad \text{--- (2)}$$

<u>Now Put the coefficient of lowest Power of x i.e:- x^{m-1} to zero:-</u>

$$(2m+0-1)(m+0)\, a_0 = 0$$

$$\rightarrow (2m-1)(m)\, a_0 = 0$$

$$\Rightarrow \quad (2m-1)(m) = 0 \qquad \text{because } a_0 \neq 0$$

which is called as Indicial Eq.

$$2m-1 = 0 \qquad m = 0$$

$$m = \frac{1}{2} \qquad m = 0$$

Now $m_1 \neq m_2$ & $m_1 - m_2 = \frac{1}{2} - 0 = \frac{1}{2} \neq$ Integer.

$\Downarrow$

$\boxed{\text{Case I}}$

From Eq. ②:- Indexing.

$$\sum_{\boxed{n=0}}^{\infty} \left[2m + 2(n+1) - 1\right] (m+n+1) a_{n+1} x^{m+n}$$

Note

$$- \sum_{n=0}^{\infty} (2m+2n-3)(m+n+1) a_n x^{m+n} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (2m+2n+1)(m+n+1) a_{n+1} x^{m+n} - \sum_{n=0}^{\infty} (2m+2n-3)(m+n+1) a_n x^{m+n} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty}\left[(2m+2n+1)(m+n+1)a_{n+1} - (2m+2n-3)(m+n+1)a_n\right]x^{m+n} = 0$$

Now by Equating coeff. of x^{m+n} to zero, we get:-

$$(2m+2n+1)(m+n+1)a_{n+1} - (2m+2n-3)(m+n+1)a_n = 0$$

$$\Rightarrow (2m+2n+1)(m+n+1)a_{n+1} = (2m+2n-3)(m+n+1)a_n$$

V. Imp. $$\boxed{a_{n+1} = \frac{(2m+2n-3)\cancel{(m+n+1)}}{(2m+2n+1)\cancel{(m+n+1)}}a_n} \quad n \geq 0 \quad (3)$$

Which is the required Recurrance Relation.

Put $n=0$ in eq. (3), :— $$\boxed{a_1 = \frac{(2m-3)}{2m+1}a_0}$$

Put $n=1$ in eq. (3) :- $$a_2 = \frac{(2m-1)}{2m+3}a_1 = \frac{2m-1}{2m+3}\left[\frac{(2m-3)}{(2m+1)}a_0\right]$$

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$$\boxed{a_2 = \frac{(2m-1)(2m-3)}{(2m+3)(2m+1)}\, a_0}$$

Now Put $n=2$ in eq. ③ :- $a_3 = \frac{(2m+1)}{(2m+5)} \cdot a_2$

$$= \frac{\cancel{(2m+1)}}{(2m+5)} \left[\frac{(2m-1)(2m-3)}{(2m+3)\cancel{(2m+1)}} a_0 \right]$$

$$\boxed{a_3 = \frac{(2m-1)(2m-3)}{(2m+5)(2m+3)}\, a_0}$$

Now As we know that the complete solution of Case I is given as:-

$$\boxed{y = c_1 (y)_{m=m_1} + c_2 (y)_{m=m_2}} \quad \text{—④}$$

Trial Sol. $y = \sum_{n=0}^{\infty} a_n x^{m+n} = a_0 x^m + a_1 x^{m+1} + a_2 x^{m+2} + a_3 x^{m+3} \text{———} \infty$

Now put the values of a_1, a_2, a_3 in Trial solution, we get:-

$$y = a_0 x^m + \left[\left(\frac{2m-3}{2m+1}\right) a_0\right] x^{m+1} + \left[\frac{(2m-1)(2m-3)}{(2m+3)(2m+1)} a_0\right] x^{m+2}$$

$$+ \left[\frac{(2m-1).(2m-3)}{(2m+5)(2m+3)} a_0\right] x^{m+3} \text{———} \infty$$

Now at $m = \frac{1}{2}$:-

$$(y)_{m=\frac{1}{2}} = a_0 \sqrt{x} + \left(\frac{(1-3)}{(1+1)} a_0\right) x^{3/2} + \left(\frac{\cancel{(1-1)}(1-3)}{(1+3)(1+1)} a_0\right) x^{5/2}$$

$$+ \left(\frac{\cancel{(1-1)}(1-3)}{(1+5)(1+3)} a_0\right) x^{7/2} + \text{———} \infty$$

$$= a_0 \sqrt{x} + \frac{(-\cancel{2})}{\cancel{2}} a_0 x^{3/2} + 0 + 0 + 0$$

$$\boxed{(y)_{m=\frac{1}{2}} = a_0 \left(x^{1/2} - x^{3/2}\right)}$$

Now at $m=0$:-

$$(y)_{m=0} = a_0 x^0 + \frac{(-3)}{(1)} a_0 x^1 + \frac{(-1)(-\cancel{3})}{(\cancel{3})(1)} a_0 x^2 + \frac{(-1)(-\cancel{3})}{(5)(\cancel{3})} a_0 x^3 \text{———} \infty$$

$$\boxed{(y)_{m=0} = a_0 \left[1 - 3x + x^2 + \frac{1}{5} x^3 \text{———} \infty \right]}$$

Now from eq. ④, we get:-

$$y = c_1 (y)_{m=\frac{1}{2}} + c_2 (y)_{m=0}$$

$$= c_1 \left[a_0 \left(x^{1/2} - x^{3/2} \right) \right] + c_2 \left[a_0 \left(1 - 3x + x^2 + \frac{x^3}{5} + \text{——} \infty \right) \right]$$

$$\boxed{y = A \left(x^{1/2} - x^{3/2} \right) + B \left(1 - 3x + x^2 + \frac{x^3}{5} + \text{——} \infty \right)}$$

which is the required complete solution of given D.E. ①.

Case II :- When roots m_1 & m_2 are Equal / Identical $(m_1 = m_2)$ (45)

Trial Sol. :- $\boxed{y = \sum_{n=0}^{\infty} a_n x^{m+n}}$; Complete Sol :- $\boxed{y = c_1 (y)_{m_1} + c_2 \left(\frac{\partial y}{\partial m}\right)_{m_2}}$

↑ Note

Q.6. Solve the Differential Equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + x^2 y = 0$ in series

[UPTU, U.P – 2013]

Sol :- According to the given Diff. Eq. :-

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + x^2 y = 0 \quad \text{——— (1)}$$

which is a Homogeneous Second Order Linear Differential Eq with variable coefficients.

Here :-

$P_0(x) = x$; $P_1(x) = 1$; $P_2(x) = x^2$

Now at $x=0$:- $P_0(0) = (0) = 0 \rightarrow$ Singular Point

Now :-

$$\boxed{\lim_{x\to a} (x-a)\frac{P_1(x)}{P_0(x)}}$$

$$\Rightarrow \lim_{x\to 0} \frac{(x)\,(1)}{x} = \boxed{1} \text{ Finite Analytic}$$

$$\boxed{\lim_{x\to a} (x-a)^2 \frac{P_2(x)}{P_0(x)}}$$

$$\Rightarrow \lim_{x\to 0} \frac{\cancel{x^2}\,x(x^2)}{\cancel{x}} = \boxed{0} \text{ finite}$$

Regular Singular Point.

So $x=0$ is a Regular Singular Point of given D.E ①

By Frobenius Method :-

m can have any value

Let Trial sol:- $\quad y = \sum_{n=0}^{\infty} a_n x^{m+n}$

$$\frac{dy}{dx} = \sum_{n=0}^{\infty} (m+n)\, a_n\, x^{m+n-1}$$

$$\frac{d^2y}{dx^2} = \sum_{n=0}^{\infty} (m+n)(m+n-1)\, a_n\, x^{m+n-2}$$

Now put the values of $y, \frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ in given D.E (1):–

$$\Rightarrow x\left[\sum_{n=0}^{\infty}(m+n)(m+n-1)a_n x^{m+n-2}\right] + \left[\sum_{n=0}^{\infty}(m+n)a_n x^{m+n-1}\right] + x^2\left[\sum_{n=0}^{\infty}a_n x^{m+n}\right] = 0$$

$$\Rightarrow \sum_{n=0}^{\infty}(m+n)(m+n-1)a_n x^{m+n-1} + \sum_{n=0}^{\infty}(m+n)a_n x^{m+n-1} + \sum_{n=0}^{\infty}a_n x^{m+n+2} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty}\left[(m+n)\underline{(m+n-1)} + \underline{(m+n)}\right]a_n x^{m+n-1} + \sum_{n=0}^{\infty}a_n x^{m+n+2} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty}(m+n)(m+n\cancel{-1}\cancel{+1})a_n x^{m+n-1} + \sum_{n=0}^{\infty}a_n x^{m+n+2} = 0$$

$$\Rightarrow \boxed{\sum_{n=0}^{\infty}(m+n)^2 a_n x^{m+n-1} + \sum_{n=0}^{\infty}a_n x^{m+n+2} = 0} \quad \text{--- (2)}$$

Now by equating least power of x :- i.e x^{m-1} equal to zero :- (48)

$$(m)^2 a_0 = 0$$

$$\Rightarrow \quad m^2 = 0 \quad [\because a_0 \neq 0]$$

which is the required Indical Eq. :-

$$\boxed{m = 0, 0}$$

So $m_1 = 0$ & $m_2 = 0$

So root of indical Equal are same $m_1 = m_2$

$$\Downarrow$$

$$\boxed{\text{Case II}}$$

From Eq. (2) :-

$$\sum_{\boxed{n=0}}^{\infty} (m+n+1)^2 a_{n+1} x^{m+n} + \sum_{\boxed{n=0}}^{\infty} a_{n-2} x^{m+n} = 0$$

Note

indexing

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$$\sum_{n=0}^{\infty}\left[(m+n+1)^2 a_{n+1} + a_{n-2}\right] x^{m+n} = 0 \quad \text{--- (3)}$$

Now put the coefficient of x^{m+n} equal to zero:-

$$(m+n+1)^2 a_{n+1} + a_{n-2} = 0$$

$$(m+n+1)^2 a_{n+1} = -a_{n-2}$$

$$\boxed{a_{n+1} = \frac{-1}{(m+n+1)^2} a_{n-2}} \quad \text{--- (4)} \quad \forall\, n \geq 2$$

which is the required recurrance Relation :-

Now put $n = 2$:-

$$\boxed{a_3 = \frac{-1}{(m+3)^2} a_0}$$

Now by Equating coefficient of x^m both sides :- in eq ② :-

$$(m+1)^2 a_1 = 0$$

$$\boxed{a_1 = 0}$$ ← note?

Now by Equating coeffi. of x^{m+1} both sides, in eq. ② :-

$$(m+2)^2 a_2 = 0$$

$$\boxed{a_2 = 0}$$ ← note?

Now put $n=3$ in eq. ④ :-

$$a_4 = \frac{-1}{(m+4)^2} a_1 = \frac{-1}{(m+4)^2}(0) \Rightarrow \boxed{a_4 = 0}$$

Put $n=4$ in eq. ④ :-

$$a_5 = \frac{-1}{(m+5)^2} a_2 = \frac{-1}{(m+5)^2}(0) \Rightarrow \boxed{a_5 = 0}$$

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Put $n = 5$ in eq. (4) :-

$$a_6 = \frac{-1}{(m+6)^2} a_3 = \frac{-1}{(m+6)^2}\left[\frac{-1}{(m+3)^2} a_0\right]$$

$$\boxed{a_6 = \frac{1}{(m+6)^2 (m+3)^2} a_0}$$

Now put the values of a_1, a_2, a_3, a_4, a_5 & a_6 in Trial sol :-

$$y = \sum_{n=0}^{\infty} a_n x^{m+n}$$

$$= a_0 x^m + a_1 x^{m+1} + a_2 x^{m+2} + a_3 x^{m+3} + a_4 x^{m+4} + a_5 x^{m+5} + a_6 x^{m+6} \text{——} \infty$$

$$= x^m \left[a_0 + (0)x + (0)x^2 + \left(\frac{-1}{(m+3)^2} a_0\right) x^3 + (0) x^4 + (0) x^5 + \left(\frac{1}{(m+6)^2 (m+3)^2} a_0\right) x^6 \text{——} \infty\right.$$

$$= x^m \left[a_0 - \frac{1}{(m+3)^2} a_0 x^3 + \frac{1}{(m+6)^2 (m+3)^2} a_0 x^6 ——\infty \right]$$

$$\boxed{y = a_0 x^m \left[1 - \frac{1}{(m+3)^2} x^3 + \frac{1}{(m+3)^2 (m+6)^2} x^6 + ——\infty \right]} \quad -⑤$$

Now complete solution of case II is given as:-

$$\boxed{y = c_1 (y)_{m_1=0} + c_2 \left(\frac{\partial y}{\partial m} \right)_{m_1=0}} \quad -⑥$$

Now $(y)_{m=0} = a_0 x^0 \left[1 - \frac{1}{(3)^2} x^3 + \frac{1}{(3)^2 (6)^2} x^6 + ——\infty \right]$

$$\boxed{(y)_{m=0} = a_0 \left[1 - \frac{x^3}{(3)^2} + \frac{x^6}{(3)^2 (6)^2} + ——\infty \right]}$$

Now Partially Differentiate eq ① w.r.t m, we get :- (53)

$$\frac{\partial y}{\partial m} = a_0 (x^m \log x)\left[1 - \frac{1}{(m+3)^2}x^3 + \frac{1}{(m+3)^2(m+6)^2}x^6 + \text{---}\infty\right]$$

$$+ a_0 x^m \left[0 + \frac{2}{(m+3)^3}x^3 + \left[-2(m+3)^{-3}(m+6)^{-2} + (m+3)^{-2}(-2(m+6)^{-3})\right]x^6 + \text{---}\infty\right]$$

[By Product Rule]

$$\frac{\partial y}{\partial m} = a_0 x^m \log x\left(1 - \frac{x^3}{(m+3)^2} + \frac{x^6}{(m+3)^2(m+6)^2} \text{---}\infty\right)$$

$$+ a_0 x^m \left[\frac{2x^3}{(m+3)^3} - \frac{2x^6}{(m+3)^3(m+6)^2} - \frac{2x^6}{(m+3)^2(m+6)^3} + \text{---}\infty\right]$$

$$\left(\frac{\partial y}{\partial m}\right)_{m=0} = a_0 (1) \log x \left[1 - \frac{x^3}{(3)^2} + \frac{x^6}{(3)^2(6)^2} + \text{———} \infty \right]$$

$$+ a_0 (1) \left[\frac{2x^3}{(3)^3} - \frac{2x^6}{(3)^3(6)^2} - \frac{2x^6}{(3)^2(6)^3} + \text{———} \infty \right]$$

(54)

Now Put $(y)_{m=0}$ and $\left(\frac{\partial y}{\partial m}\right)_{m=0}$ in eq. ⑥, we get:-

$$\Rightarrow y = c_1 \left[a_0 \left(1 - \frac{x^3}{(3)^2} + \frac{x^6}{(3)^2(6)^2} + \text{———} \infty \right) \right]$$

$$+ c_2 \left[a_0 \log x \left(1 - \frac{x^3}{(3)^2} + \frac{x^6}{(3)^2(6)^2} + \text{———} \infty \right) \right.$$

$$\left. + a_0 \left(\frac{2x^3}{(3)^3} - \frac{2x^6}{(3)^3(6)^2} - \frac{2x^6}{(3)^2(6)^3} \text{———} \infty \right) \right]$$

$$y = A\left(1 - \frac{x^3}{(3)^2} + \frac{x^6}{(3)^2(6)^2} \text{———} \infty\right)$$

$$+ B\left[\log x\left(1 - \frac{x^3}{(3)^2} + \frac{x^6}{(3)^2(6)^2} \text{———} \infty\right)\right.$$

$$\left. + \left(\frac{2x^3}{(3)^3} - \frac{2x^6}{(3)^3(6)^2} - \frac{2x^6}{(3)^2(6)^3} \text{———} \infty\right)\right]$$

$$\boxed{\begin{aligned} y &= (A + B\log x)\left(1 - \frac{x^3}{(3)^2} + \frac{x^6}{(3)^2(6)^2} \text{———} \infty\right) \\ &\quad + 2B\left[\frac{x^3}{(3)^3} - \frac{1}{(3)^2(6)^2}\left[\frac{1}{3} + \frac{1}{6}\right)x^6 + \text{———} \infty\right] \end{aligned}}$$

which is the required complete solution of given D.E (1) B

Case - III :- Values of roots m_1 & m_2 are distinct (i.e. $m_1 \neq m_2$) and they Differ by an Integer (i.e $m_1 - m_2$ = Integer)

Trial Sol: – $\boxed{y = \sum_{n=0}^{\infty} a_n x^{m+n}}$; Complete Sol. :– $\boxed{y = c_1 (y)_{m_1} + c_2 \left(\frac{\partial y}{\partial m}\right)_{m_2} \text{ if } m_1 > m_2}$

Q.7. Solve the Bessel's Differential Eq. of order 2 in series.

OR

Solve the Bessel's Diff. Eq of order 2 in series

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-4)y = 0$$

OR

Using Frobenius method solve the Diff. Eq. $x^2y'' + xy' + (x^2-4)y = 0$ about the point $x = 0$

Sol:- As we know that, Bessel's Differential Eq. of order n is (57)

written as:- $$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-n^2)y = 0$$

Here order $n=2$:-

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-4)y = 0 \quad \text{—} (1)$$

Here:- $P_0(x) = x^2$; $P_1(x) = x$; $P_2(x) = x^2-4$

Now at Point $x=0$:- $P_0(0) = (0)^2 = 0 \rightarrow$ Singular Point

$$\boxed{\lim_{x\to a} (x-a)\frac{P_1(x)}{P_0(x)}}$$

$$\lim_{x\to 0} \frac{\cancel{(x)}\cancel{(x)}}{\cancel{x^2}} = \boxed{1}$$

Finite

Analytic

$$\boxed{\lim_{x\to a} (x-a)^2\frac{P_2(x)}{P_0(x)}}$$

$$\lim_{x\to 0} \frac{\cancel{(x^2)}(x^2-4)}{\cancel{x^2}} = \boxed{-4}$$

Finite

$\rightarrow$ Regular singular Point