COMPLEX VARIABLE - I

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COMPLEX NUMBERS

A complex number is a number consisting of a Real and Imaginary part.

It can be written in the form

$$z = \underbrace{a}_{\text{Real Part}} + \underbrace{bi}_{\text{Imaginary Part}}$$

THE POWERS OF i

If $i = \sqrt{-1}$, then :

$i^2 = -1 \qquad i^3 = -i \qquad i^4 = 1$

$i^6 = -1 \qquad i^7 = -i \qquad i^8 = 1$

For i^n ... divide n by 4...

- *If n is evenly divisible by 4 then $i^n = 1$*
- *If the remainder is 1 then $i^n = i$*

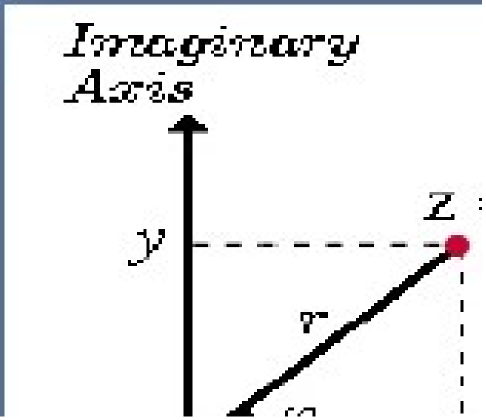
COMPLEX CONJUGATE

- The **COMPLEX CONJUGATE** of a complex r
$z = x + iy$, denoted by z^* , is given by

$$z^* = x - iy$$

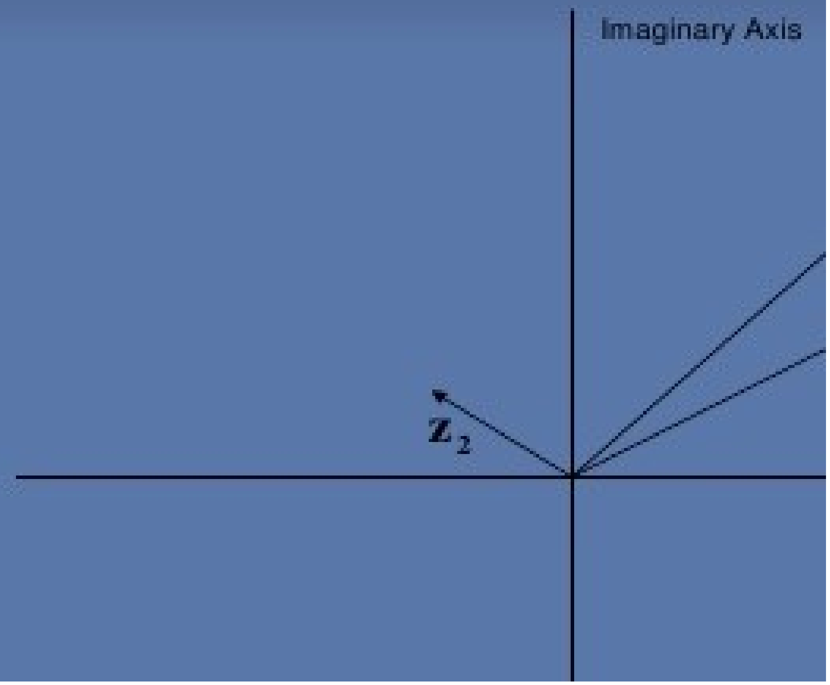
- The **Modulus** or *absolute value*
is defined by

$$z = \sqrt{x^2 + y^2}$$



ADDITION OF COMPLEX NUM

$$(a+bi)+(c+di)=(a+c)+(b+d$$



EXAMPLE

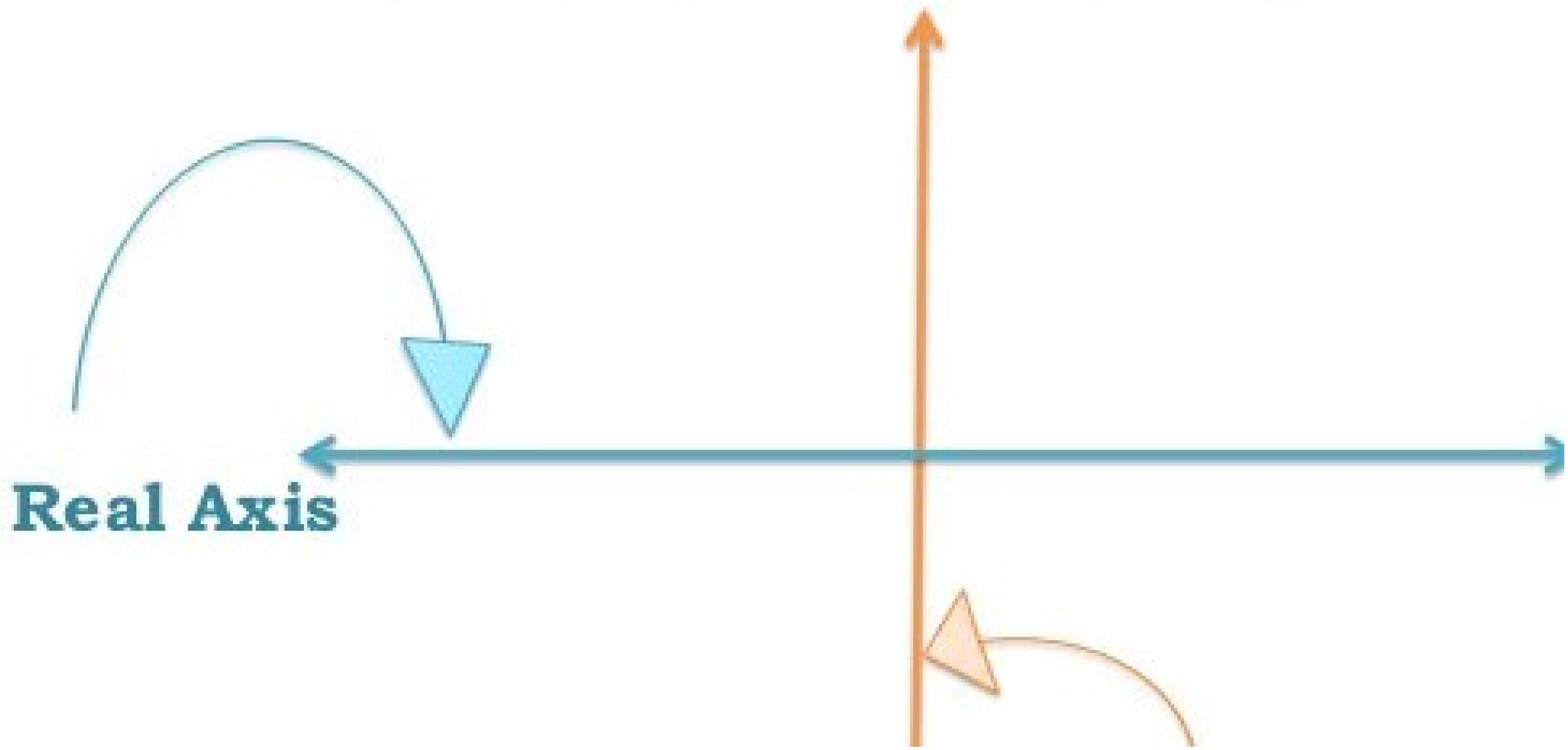
$$(2+3i)+(1+5i)$$

$$=(2+1)+(3+5)i$$

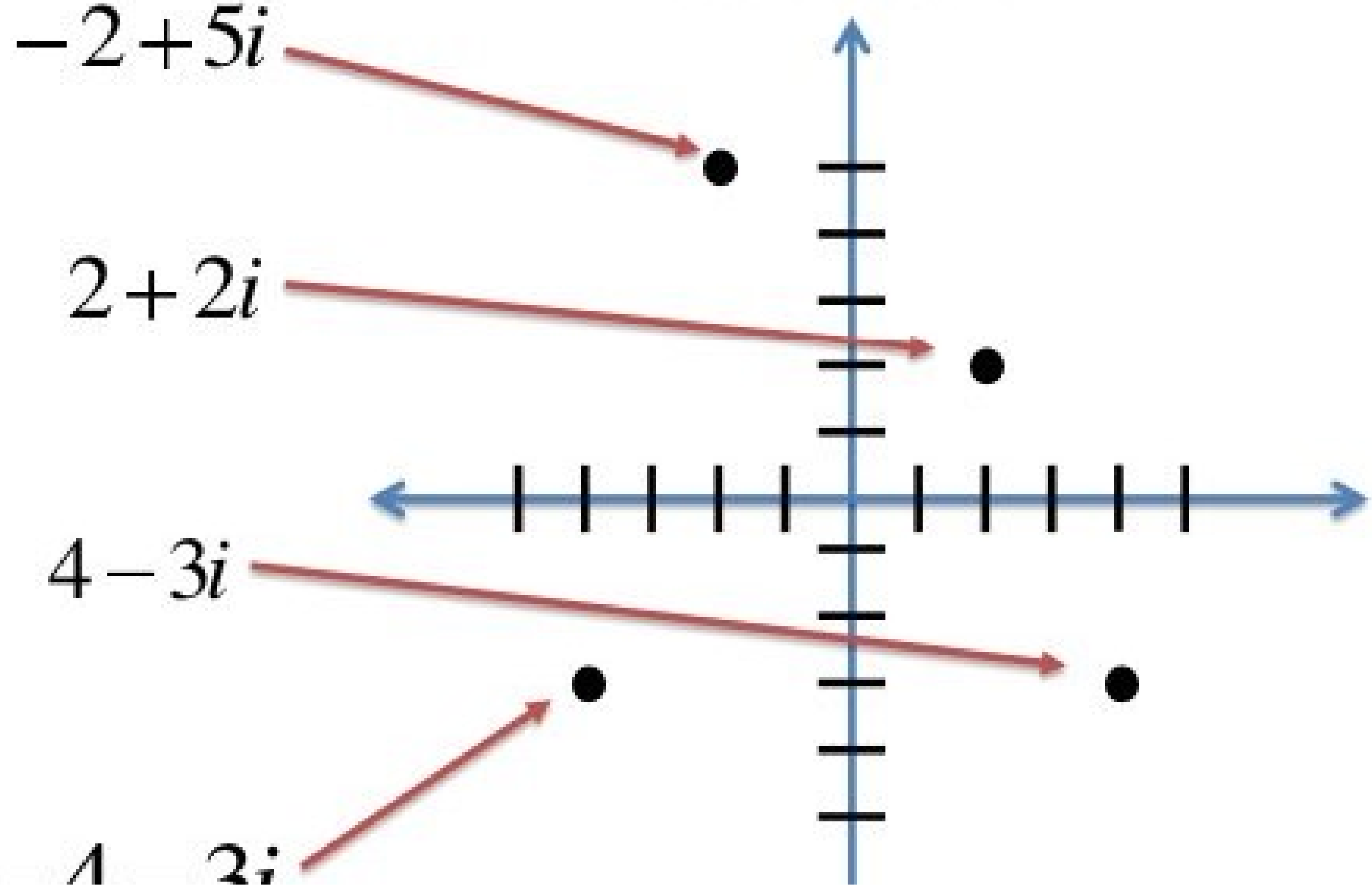
$$=3+8i$$

THE COMPLEX PLANE

- We modify the familiar coordinate system by calling tl
  axis the **real axis** and the vertical axis the **imaginary**
- Each complex number $a + bi$ determines a unique pos
  with initial point (0, 0) and terminal point (a, b).



GRAPHING IN THE COMPL PLANE



Absolute Value of a Complex Numbe

- The distance the complex number is from the origin complex plane.
- If you have a complex number $(a+bi)$

the absolute value can be found using:

$$\sqrt{a^2+b^2}$$

Examples

1. $|-2+5i|$

$$=\sqrt{(-2)^2+(5)^2}$$

$$=\sqrt{4+25}$$

2. $|-6i|$

$$=\sqrt{(0)^2+}$$

$$=\sqrt{0+}$$

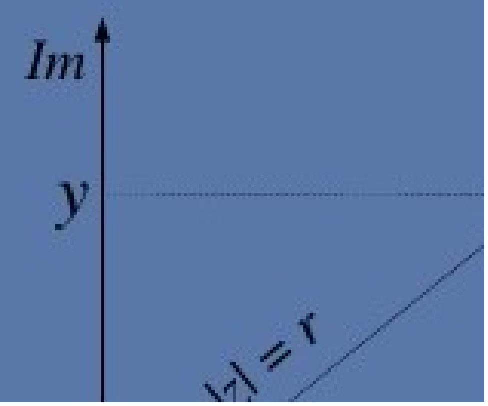
Argument of Complex Number:

Geometrically, $|z|$ is the distance of the point z from t
origin while θ is the directed angle from the positi
axis to OP in the above figure.

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

θ is called the **argument** of z and is denoted by arg z. Thus,

$$\theta = \arg z = \tan^{-1}\left(\frac{y}{x}\right) \qquad z \neq 0$$



Expressing Complex Number in Polar Form

$$x = r\cos\theta \qquad y = r\sin\theta$$

So any complex number, x + iy,

$$x + iy = r(\cos\theta + i\sin\theta)$$

Example

A complex number, $z = 1 - j$

has a magnitude

$$|z| = \sqrt{(1^2 + 1^2)} = \sqrt{2}$$

and argument :

$$\angle z = \tan^{-1}\left(\frac{-1}{1}\right) + 2n\pi = \left(-\frac{\pi}{4} + 2n\right.$$

Hence its principal argument is : $Arg\ z = -\frac{\pi}{4}$ rac

Hence in polar form :

$$z = \sqrt{2}e^{-j\frac{\pi}{4}} = \sqrt{2}\left(\cos\frac{\pi}{4} - j\sin\frac{\pi}{4}\right)$$

EULER FORMULA – AN ALTERNATE POLAR FORM

The polar form of a complex number can be rewritten as :

$$z = r(\cos\theta + j\sin\theta) = x + jy$$

$$= re^{j\theta}$$

This leads to the complex exponential function :

$$e^{z} = e^{x+jy} = e^{x}e^{jy}$$

$$= e^{x}\left(\cos y + j\sin y\right)$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

EULER FORMULA

- Remember the well-known Taylor Expansions :

$$\cos\theta = 1 - \frac{1}{2}\theta^2 + \frac{1}{4!}\theta^4 - \frac{1}{6!}\theta^6 + \cdots$$

Also

Now, let's look at $e^{i\theta}$. The power series expansion of this function is g

EULER FORMULA

Now, let's look at $e^{i\theta}$. The power series expansion of this function is g

$$e^{i\theta} = 1 + i\theta + \frac{1}{2}(i\theta)^2 + \frac{1}{3!}(i\theta)^3 + \frac{1}{4!}(i\theta)^4 + \frac{1}{5!}(i\theta)^5 + \cdots$$

$$= 1 + i\theta - \frac{1}{2}\theta^2 - i\frac{1}{3!}\theta^3 + \frac{1}{4!}\theta^4 + i\frac{1}{5!}\theta^5 + \cdots$$

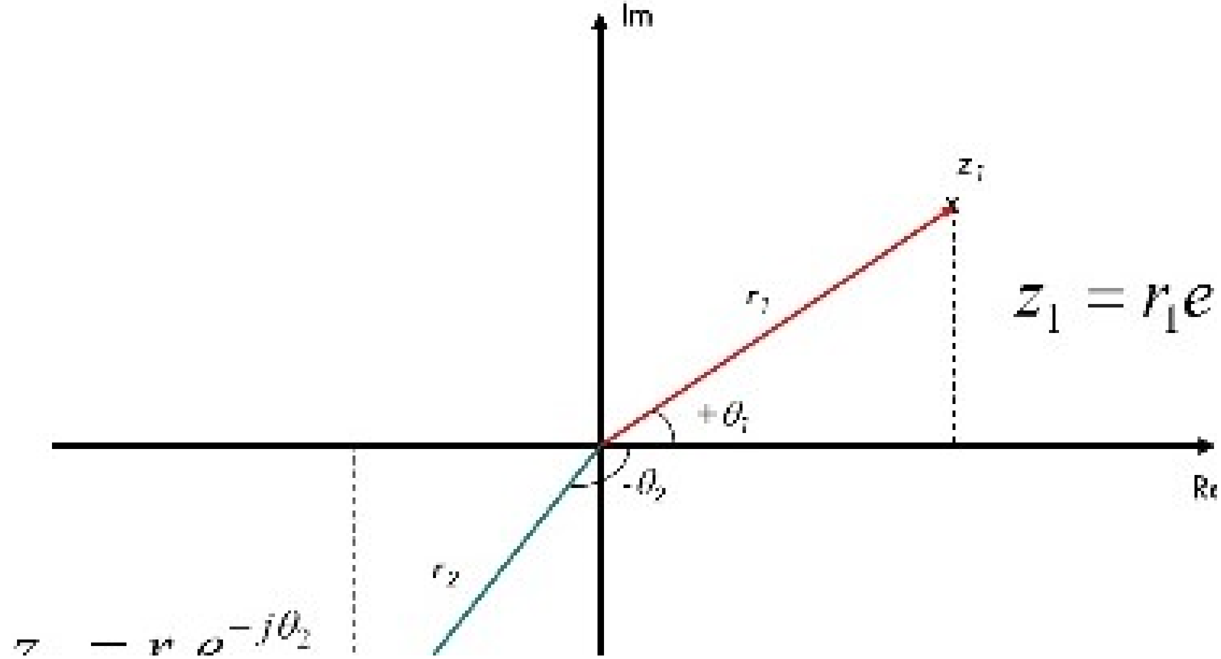
(Now group terms—looking for sin and cosine)

$$= \left(1 - \frac{1}{2}\theta^2 + \frac{1}{4!}\theta^4 - \cdots\right) + \left(i\theta - i\frac{1}{3!}\theta^3 + i\frac{1}{5!}\theta^5 + \cdots\right)$$

$$= \left(1 - \frac{1}{2}\theta^2 + \frac{1}{4!}\theta^4 - \cdots\right) + i\left(\theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 + \cdots\right)$$

$$= \cos\theta + i\sin\theta$$

GRAPHIC REPRESENTATION



De Moivre's Theo

De Moivre's Theorem is the theorem
shows us how to take complex numbe
power easily.

Let r(cos Φ+*i*sin Φ) be a **complex nu**

Thanks