VECTOR ANALYSIS

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4. Vector Addition
5. Vector Multiplication
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    -- Scalar Multiplication of two Vectors
    -- Vector Multiplication of two Vectors

What

A scalar is a physical quantit
has only a magnitude.

Examples:

- Mass
- Temperatur
- Length
- Volume

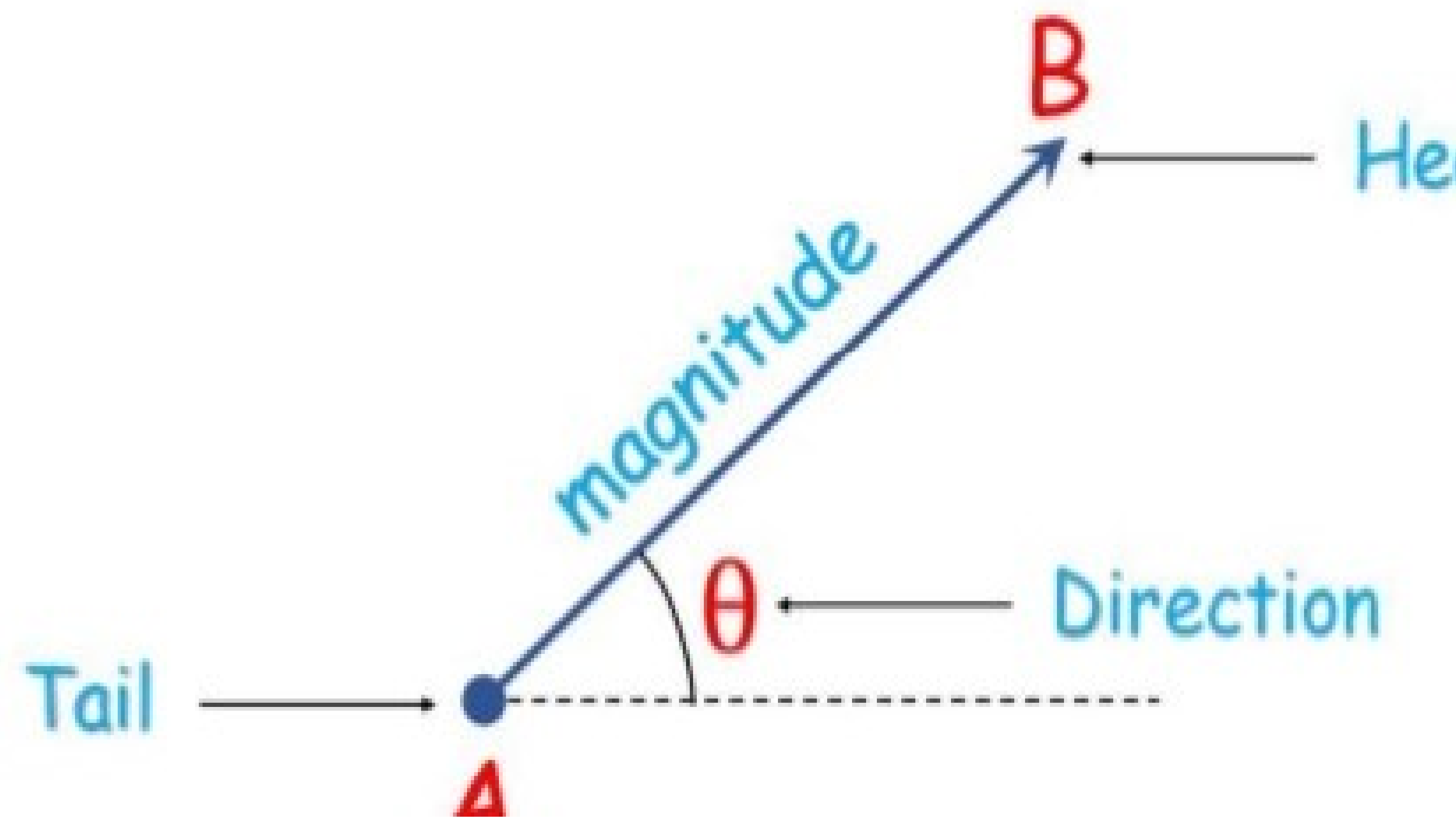
What

A vector is a physical quantity t
both a magnitude and a direc

Examples:

- Position
- Displacement
- Acceleratio
- Momentum

Representation of a vector



Types of Vectors

Unit vectors

- A given vector can be expressed as a p its magnitude and a unit vector.
- For example $\vec{A}$ may be represented as,

$$\vec{A} = A\,\hat{A}$$

A = magnitude of $\vec{A}$

Parallel Vectors

Two vectors are said to be parallel v
they have same direction.

Anti-parallel Vectors

Two vectors are said to be anti-paralle
if they are in opposite directior

Equal Vectors

Two parallel vectors are said to be equa
if they have same magnitude.

Negative Vectors

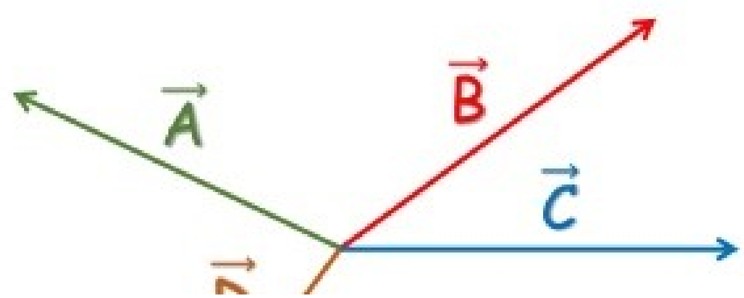
Two anti-parallel vectors are said to be
vectors, if they have same magnitu

Collinear Vectors

Two vectors are said to be collinear if they act along a same line.

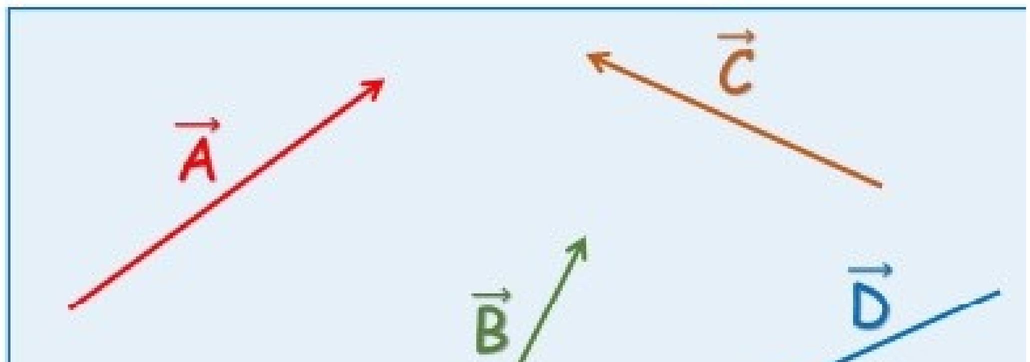
Co-initial Vectors

Two or more vectors are said to be c vectors, if they have common initia



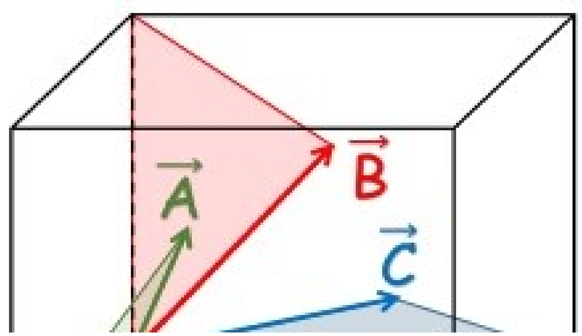
Coplanar Vectors

Three or more vectors are said to be
vectors, if they lie in the same pl



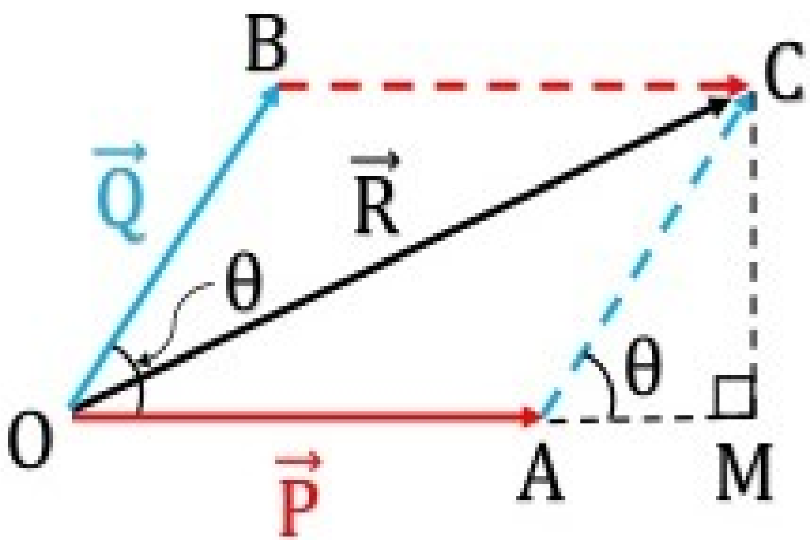
Non-coplanar Vectors

Three or more vectors are said to be non
vectors, if they are distributed in sp



Vecto
Additi

Magnitude of Resultant



In ΔOCM,

$$OC^2 = OM^2 + CM^2$$

$$OC^2 = OA^2 + 2OA \times$$

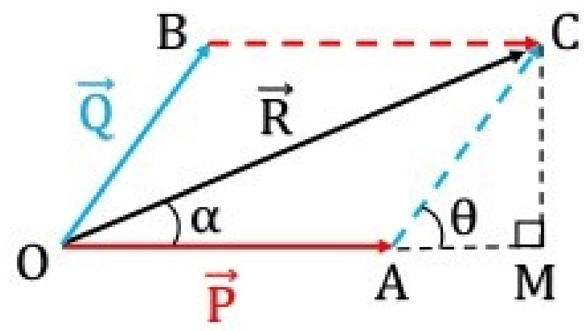
In ΔCAM,

$$\cos\theta = \frac{AM}{AC} \quad \Rightarrow \quad A$$

$$OC^2 = OA^2 + 2OA \times$$
$$+ AC^2$$

$$R^2 = P^2 + 2P \times Q\,co$$

Direction of Resultant



In ΔCAM,

$\sin\theta = \frac{CM}{AC} \Rightarrow CM = AC \sin\theta$

In ΔOCM,

$\tan\alpha = \frac{CM}{OM}$

$\tan\alpha = \frac{CM}{OA+A}$

$\tan\alpha = \frac{AC}{OA+A}$

Case I - Vectors are parallel (θ

SPECIAL CASE

Magnitude:

$R = \sqrt{P^2 + 2PQ\cos 0° + Q^2}$

$R = \sqrt{P^2 + 2PQ + Q^2}$

$R = \sqrt{(P + Q)^2}$

Direction:

$\tan\alpha = \frac{Q \sin}{P + Q\, cc}$

$\tan\alpha = \frac{0}{P + Q} =$

SPECIAL CASE

Case II - Vectors are perpendicular (

Magnitude:

$$R = \sqrt{P^2 + 2PQ\cos 90^\circ + Q^2}$$

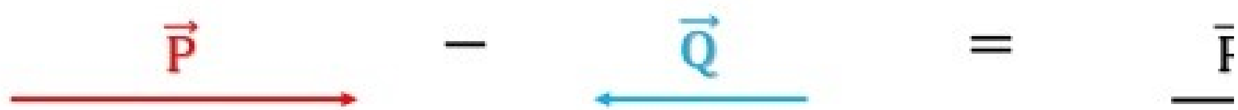
$$R = \sqrt{P^2 + 0 + Q^2}$$

Direction:

$$\tan\alpha = \frac{Q\sin 90}{P + Q\cos 9}$$

SPECIAL CASE

Case III - Vectors are anti-parallel (



Magnitude:

$$R = \sqrt{P^2 + 2PQ\cos 180^\circ + Q^2}$$

$$R = \sqrt{P^2 - 2PQ + Q^2}$$

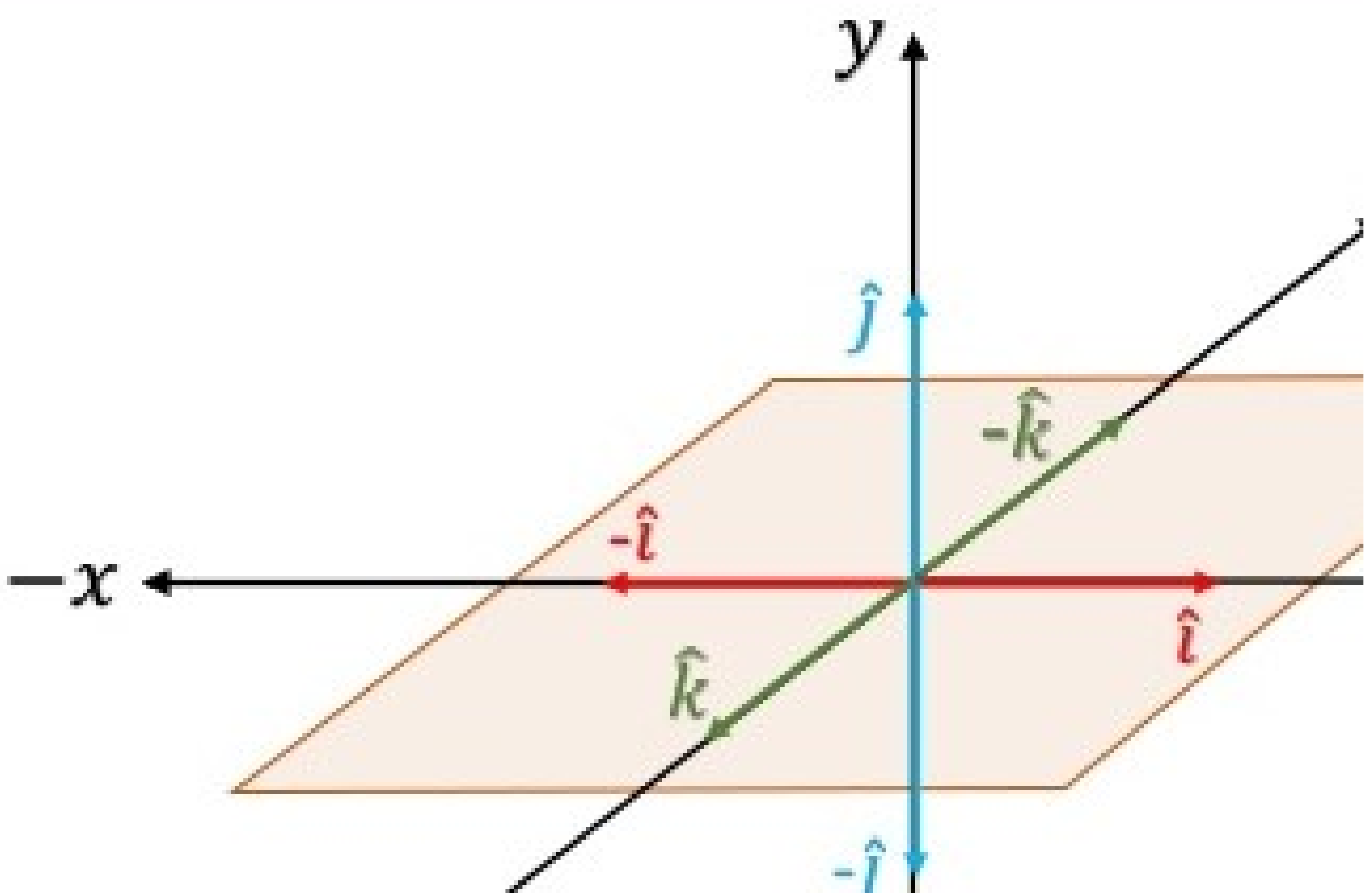
$$R = \sqrt{(P - Q)^2}$$

Direction:

$$\tan\alpha = \frac{Q\sin 1}{P + Q\cos}$$

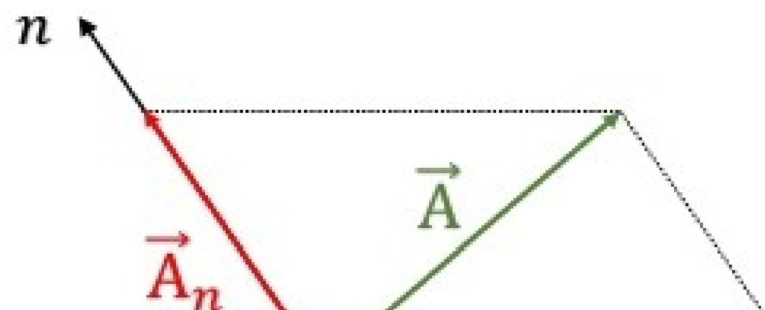
If $P > Q$: $\alpha =$

Cartesian unit vect

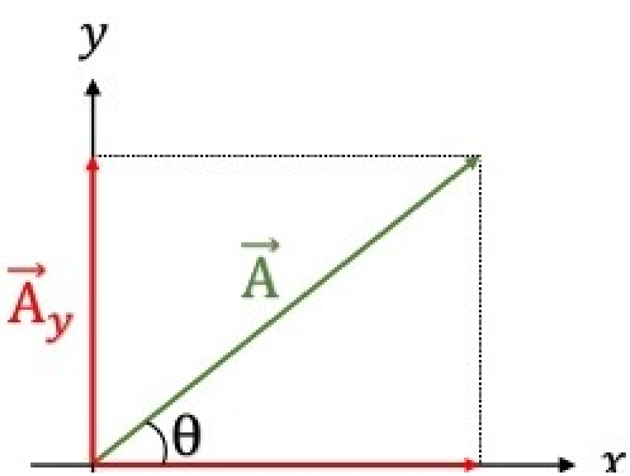


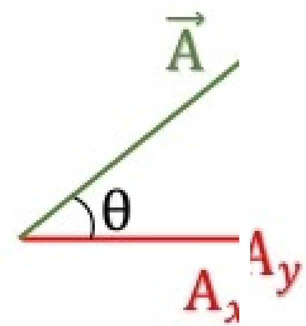
Resolution of a Vector

It is the process of splitting a vector into two or
in such a way that their combined effect is same
the given vector.



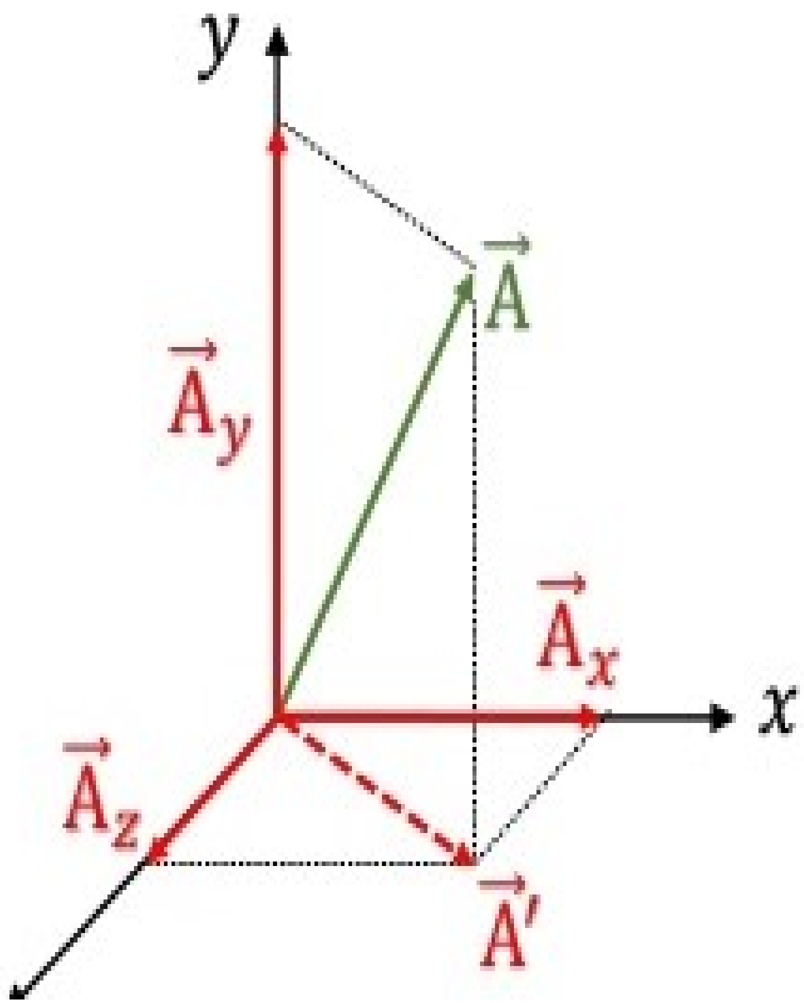
Rectangular Components of 2D





$$\sin\theta = \frac{A_y}{A} \Rightarrow A_y$$

Rectangular Components of 3D Ve



$$\vec{A} = \vec{A}' + \vec{A}_y$$

$$\vec{A} = \vec{A}_x + \vec{A}_z$$

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

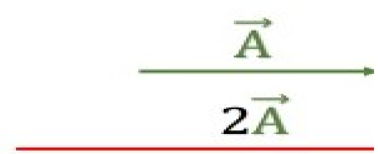
$$\vec{A} = A_x\hat{i} + A_y$$

Multiply

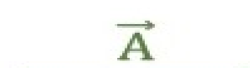
A) Multiplying a vector by a scala

- If we multiply a vector $\vec{A}$ by a scalar s, new vector.
- Its magnitude is the product of the m of $\vec{A}$ and the absolute value of s.

If s is positive:



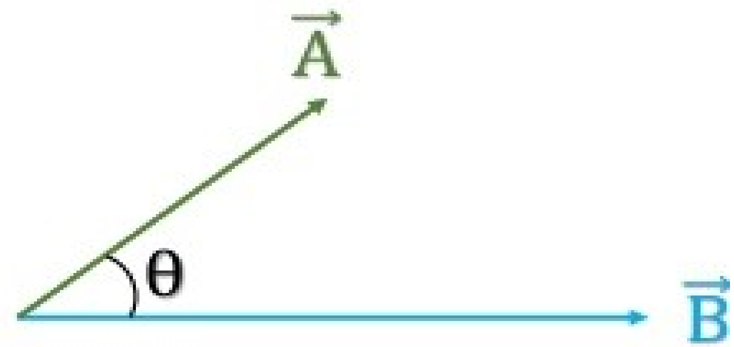
If s is negative:



B) Multiplying a vector by a vect

- There are two ways to multiply a ve vector:
- The first way produces a scalar qua called as scalar product (dot product).

Scalar product



Examples of scalar product

$$W = \vec{F} \cdot \vec{s}$$

$$W = Fs\cos\theta$$

W = work done

F = force

$$P = \vec{F}$$

$$P = Fv$$

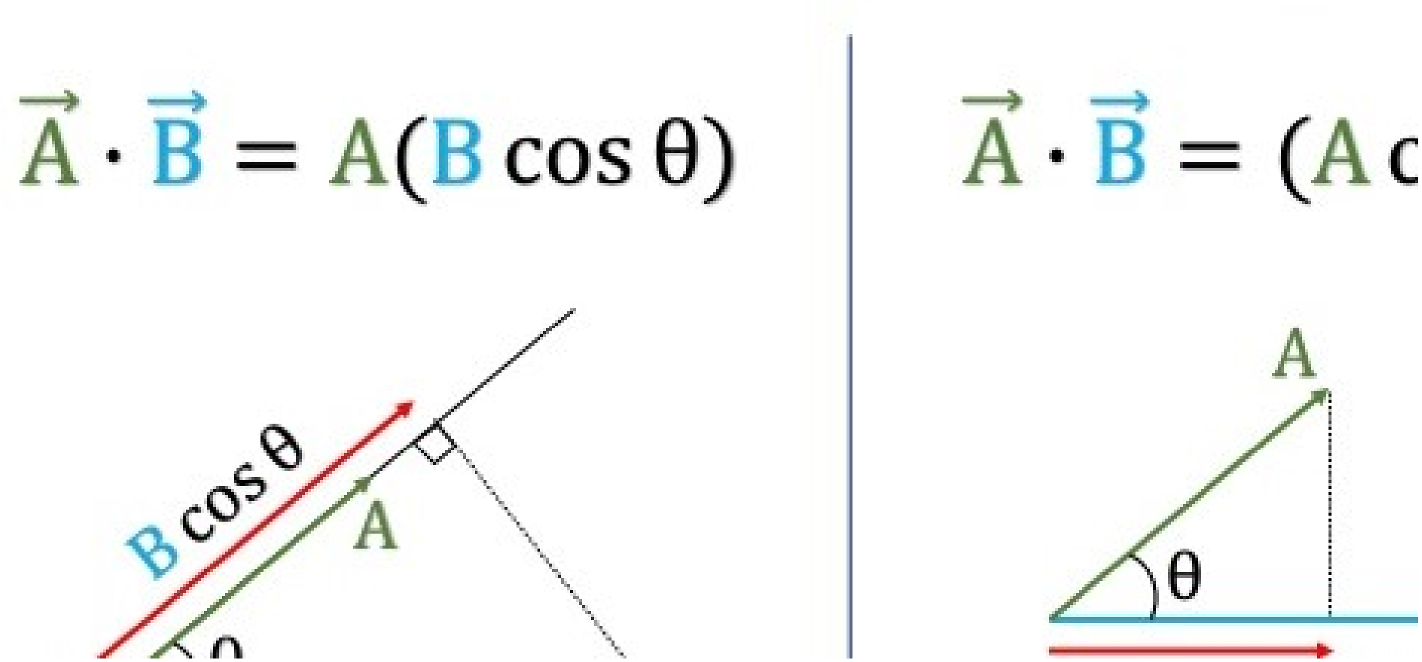
P = pow

F = for

Geometrical meaning of Scalar dot p

A dot product can be regarded as the of two quantities:

1. The magnitude of one of the vectors



Properties of Scalar product

1

The scalar product is comm

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\vec{B} \cdot \vec{A} = BA\cos\theta$$

2

The scalar product is distribu
addition.

3

The scalar product of two perpe
vectors is zero.

$$\vec{A} \cdot \vec{B} = AB\cos 90°$$

4

The scalar product of two paralle
is maximum positive.

$$\vec{A} \cdot \vec{B} = AB\cos 0°$$

5

The scalar product of two anti
vectors is maximum ne

$$\vec{A} \cdot \vec{B} = AB\cos 180$$

6

The scalar product of a vector wi
is equal to the square of its mag

$$\vec{A} = AA\cos 0\,^{\circ}$$

7

The scalar product of two same unit
one and two different unit vectors

$$\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = (1)(1)\cos($$

Calculating scalar product using com

Let us have

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

then

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\vec{A}\cdot\vec{B} = A_x\hat{i}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_xB_x\hat{i}\cdot\hat{i} + A_xB_y\hat{i}\cdot\hat{j}$$

$$+ A_yB_x\hat{j}\cdot\hat{i} + A_yB_y\hat{j}\cdot$$

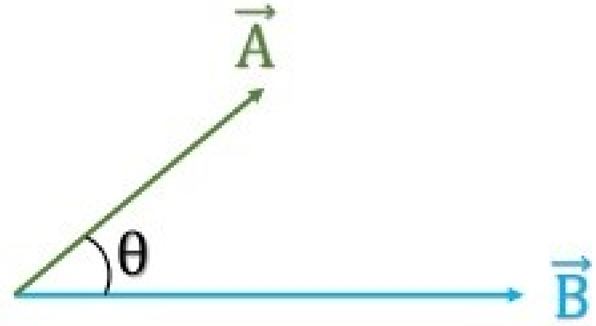
$$+ A_zB_x\hat{k}\cdot\hat{i} + A_zB_y\hat{k}\cdot$$

$$= A_xB_x(1) + A_xB_y(0)$$

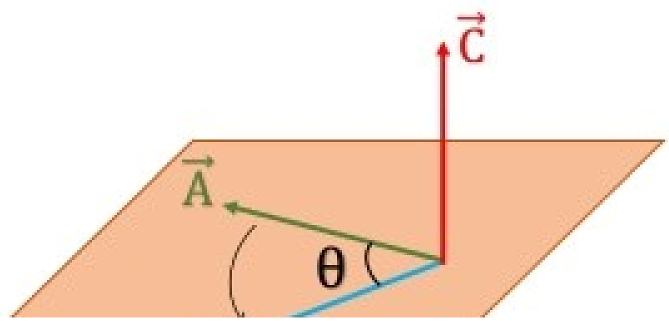
$$+ A_yB_x(0) + A_yB_y($$

$$+ A_zB_x(0) + A_zB_y($$

Vector product



Right hand rule



Examples of vector product

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = rF \sin\theta \, \hat{n}$$

τ = torque

r = position

$$\vec{L} = \vec{r} \times$$

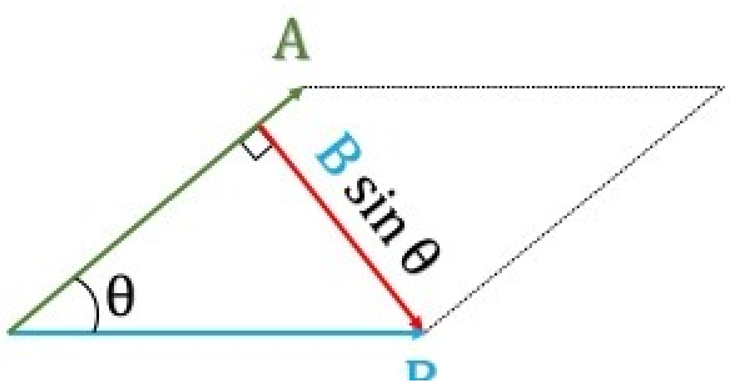
$$\vec{L} = rp \sin$$

L = angular m

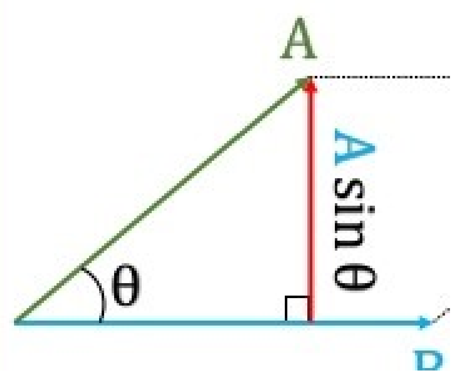
r = position

Geometrical meaning of Vector pr

$$|\vec{A} \times \vec{B}| = A(B \sin\theta)$$



$$|\vec{A} \times \vec{B}| = (A$$



Properties of Vector p

1

The vector product is anti-comı

$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$$

$$\vec{B} \times \vec{A} = BA \sin\theta\, (-\hat{n}) = -A$$

2

The vector product is distribut
addition.

3

The magnitude of the vector produ
perpendicular vectors is maxin

$$|\vec{A} \times \vec{B}| = AB \sin 90\,°$$

4

The vector product of two paralle
is a null vec

$\vec{A} \times \vec{B} = AB\, \mathrm{si}$

5

The vector product of two ant
vectors is a null vector

$\vec{A} \times \vec{B} = AB \sin 180° \, \hat{n}$

6

The vector product of a vector wi
is a null vector.

$\vec{A} \times \vec{A} = AA \sin 0° \, \hat{n}$

7

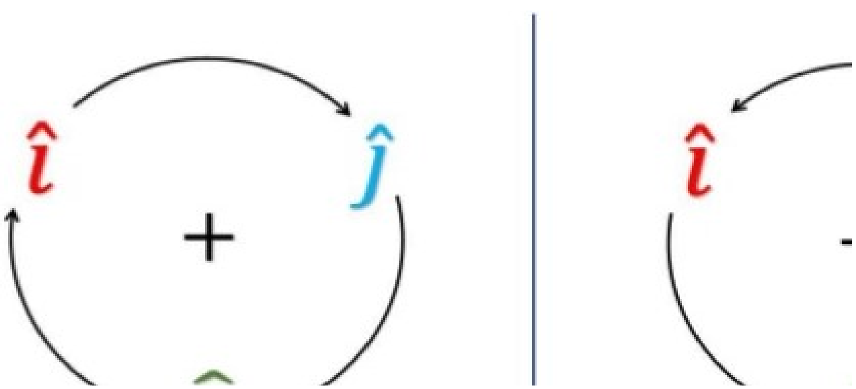
The vector product of two sa
vectors is a null vector

$$\hat{\imath}\times\hat{\imath}=\hat{\jmath}\times\hat{\jmath}=\hat{k}\times\hat{k}$$

8

The vector product of two differ
vectors is a third unit vecto

$\hat{\imath}\times\hat{\jmath}=\hat{k}$ | $\hat{\jmath}\times\hat{\imath}=$



Calculating vector product using com

Let us have

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

then

$$\vec{A} \times \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$\vec{A} \times \vec{B} = A_x\hat{\imath} \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$\vec{A} \times \vec{B} = \hat{\imath}\,(A_yB_z - A_zB_y) - \hat{\jmath}\,(A_xB$$

$$+ \hat{k}\,(A_xB_y - A_yB_x)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} \\ A_x & A_y \end{vmatrix}$$

$$= A_xB_x\hat{\imath} \times \hat{\imath} + A_xB_y\hat{\imath} \times \hat{\jmath}$$
$$+ A_yB_x\hat{\jmath} \times \hat{\imath} + A_yB_y\hat{\jmath} \times \hat{\jmath}$$
$$+ A_zB_x\hat{k} \times \hat{\imath} + A_zB_y\hat{k} \times$$

$$= A_xB_x(\vec{0}) + A_xB_y(\hat{k}) +$$
$$+ A_yB_x(-\hat{k}) + A_yB_y(\vec{0}$$
$$+ A_zB_x(\hat{\jmath}) + A_zB_y(-\hat{\imath})$$

THANK YOU

ALL THE BEST

https://www.slideshare.net/KhanSaif2/1-scalars-vectors

https://www.slideshare.net/KunjPatel4/vector-calculus-and-linear-algebra