

7

Fitting of Trend from a given set of data.

Example (1) following data is given. Fit a trend.

Year: 1961 1962 1963 1964 1965

Production: 6 7 10 6 12
(in tonnes)

Solⁿ: Let the trend line be

$$Y = a + bx \rightarrow (1)$$

To determine a and b the normal equations are

$$\sum Y_t = na + b \sum X_t \rightarrow (2)$$

$$\sum X_t Y_t = a \sum X_t + b \sum X_t^2 \rightarrow (3)$$

Year t	Y_t	$X_t = t - 1963$	X_t^2	Y_t^2	$X_t Y_t$
1961	6	-2	4	36	-12
1962	7	-1	1	49	-7
1963	10	0	0	100	0
1964	6	1	1	36	6
1965	12	2	4	144	24
	$\sum Y_t = 41$	$\sum X_t = 0$	$\sum X_t^2 = 10$	$\sum Y_t^2 = 365$	$\sum X_t Y_t = 11$

After substituting equations (2) and (3) become

$$41 = 5a + b \times 0 \quad \text{and} \quad 11 = a \times 0 + 10b$$

$$\text{or, } 41 = 5a$$

$$\text{or } 10b = 11$$

$$\text{or, } a = \frac{41}{5} = 8.2$$

$$\text{or } b = \frac{11}{10} = 1.1$$

Putting the values of a and b in equation ①

We get the required trend line as

$$Y_t = 8.2 + 1.1X_t.$$

Note: If n , the number of time series values is odd, then the transformation is

$$x = \frac{t - \text{middle value}}{\text{Interval } (h)} \rightarrow \text{②}$$

If n is even then

$$x = \frac{t - (\text{A.M. of two middle values})}{\frac{1}{2} \text{Interval } (h)}$$

Example 2 Fit a trend line to the following data by the least square method.

Year :	1975	1977	1979	1981	1983
Prod : (in '000 tons)	18	21	23	27	16

Estimate the production in 1980 and 1985.

$$\boxed{\text{Ans: } Y = 21 + 0.2X} \quad \text{--- (A)}$$

2nd Part:

$$\text{Here, } X = \frac{t - 1979}{2}$$

The estimated $\overset{2}{\text{prod}}$ in 1980 is obtained on putting $X = \frac{t - 1979}{2} = \frac{1980 - 1979}{2} = \frac{1}{2} = 0.5$ in eqn (A).

$$\text{Thus } Y_{1980} = 21 + 0.2 \times 0.5 = 21 + 0.1 = 21.1 \text{ ('000 tons)}$$

The estimated production in 1985 is obtained

$$\text{on putting } X = \frac{t - 1979}{2} = \frac{1985 - 1979}{2} = \frac{6}{2} = 3$$

eqn (A)

$$\text{Thus, } Y_{1985} = 21 + 0.2 \times 3 = 21 + 0.6 = 21.6 \text{ ('000 tons)}$$