Partial Differential Equations

Introduction:

The differential eqn which involve two or more independent variables and partial derivatives of the dependent variable(s) with respect to those independent variables are called partial differential eqn. Thus an eqn relating partial derivatives is called partial differential eqn.

The order of a p.d. eqn is the highest ~~height~~ partial differential co-efficient appearing in it. The degree of such eqn is the greatest exponent of the height

In the theory of p.d. eqn a variable z is a function of more than one independent variable. In case there are n independent variables, we take those as ~~...~~ $x_1, x_2 \cdots x_n$. However, the study is generally contained to when z is a function of variables x and y or w.r.to

$$z = f(x, y)$$

We adopt the following notation throughout the study of p.d. eqn:

$$p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}, \quad r = \frac{\partial z}{\partial z}$$

$$s = \frac{\partial^2 z}{\partial x \partial y}, \quad t = \frac{\partial^2 z}{\partial y^2}$$

Sometimes p.d. are also denoted by making use of suffixes then ~~on~~ w.r.to we write

$$u_x = \frac{\partial u}{\partial x}, \quad u_y = \frac{\partial u}{\partial y}, \quad u_{xy} = \frac{\partial u}{\partial x \partial y},$$

$$u_{yy} = \frac{\partial^2 u}{\partial y^2}$$

A first order p.d eqn with z as dependent variable and x, y as independent variables can be written in symbolic form as $f(x, y, z, p, q) = 0$

A common form of 2nd order p.d. eq^n in two independent variables x, y is

$$Rr + Ss + Tt + Pp + Qq + f(x, y, z) = 0$$

where R, S, T, P, Q are function of x, y, and f is a prescribed function of x, y, z

The general form of 2nd order p.d. eq^n is $F(x, y, z, p, q, r, s, t) = 0$

Again partial differential eq^n may be classified into two broad classes linear p.d. eq^n and non linear p.d. eq^n.

The degree of a p.d. eq^n is the exponent of the highest order. Such an eq^n is called linear if it is of the 1st degree in the dependent variable and its partial derivatives that is the power and/or products of the dependent variables and gives partial derivatives must be absent. ~~and~~ An eq^n which is not linear is called a non-linear partial differential equation

Ex

$$\frac{\partial^2 z}{\partial x \partial t} + \frac{\partial z}{\partial t} = 0 \longrightarrow$$ linear partial differential equation of two independent variables x and t

$$6\left(\frac{\partial z}{\partial x}\right)^3 + \frac{\partial z}{\partial t} = 0 \longrightarrow$$ non linear p.d.eqn of order one (of two independent variables x and t)

A partial differential eqn is said to be quasi-linear if it is linear in the highest order derivatives of the unknown function

Ex $$\frac{\partial u}{\partial t} + cu\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0 \rightarrow \text{K.d.V eq}^n.$$

Article

origin of P.D. eqn

The p.d. eqn may be obtained in the following ways

I. Elimination of arbitrary constant

II. Elimination of arbitrary function.

1. Elimination of Arbitrary constant.

Let there be any function

$$f(x, y, z, a, b) = 0 \longrightarrow (i)$$

where a, b are arbitrary constant.

Differentiating (i) partially with respect to x and y in turn respectively we get

$$\frac{\partial f}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial x} = 0 \longrightarrow (ii)$$

$$\frac{\partial f}{\partial x}\frac{\partial x}{\partial y} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial y} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial y} = 0 \longrightarrow (iii)$$

Here $\frac{\partial y}{\partial x} = 0 = \frac{\partial x}{\partial y}$, Since x and y are independent and $\frac{\partial x}{\partial x} = 1 = \frac{\partial y}{\partial y}$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

So the eqn (ii) and (iii) become

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial z} \cdot p = 0 \longrightarrow (iv)$$

$$\frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} \cdot q = 0 \longrightarrow (v)$$

In general, the arbitrary constants a and b may be ~~el~~ eliminated from the eqn (i) ~~(iii)~~ (iv) and (v), then we get a partial differential eqn of first order

$$f(x, y, z, p, q) = 0 \longrightarrow \text{(vi)}$$

The eqn (vi) is often called the eliminant of the eqn (i)