$$n = \frac{2\pi}{T} = \frac{k\sqrt{m_1+m}}{a^{3/2}}$$

Let the unit of distance be $(-x_1+x_2)$ [i.e $a=1$]

and let the unit of time be such as to

make $k=1$.

$n=1$, [$\because k=1$ & $m_1+m_2=1$]

$\therefore$ the eqn of motion becomes.

$$\ddot{x} - 2\dot{y} = x - \frac{(1-m)}{\rho_1^3}(x-x_1) - \frac{m}{\rho_2^3}(x-x_2) \rightarrow (6)$$

$$\ddot{y} + 2\dot{x} = y - \frac{(1-m)}{\rho_1^3}y - \frac{m}{\rho_2^3}y \rightarrow (7)$$

$$\ddot{z} = -\frac{(1-m)}{\rho_1^3}z - \frac{m}{\rho_2^3}z \rightarrow \boxed{(8)} ?$$

The general problem of determining the

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motion of the infinitesimal mass is, therefore

one requiring six Integrals for its complete

solutions.

Now, let us consider a function U defined

by $U = \frac{1}{2}(x^2+y^2) + \frac{1-m}{\rho_1} + \frac{m}{\rho_2} \rightarrow (9)$

where $\rho_1^2 = (x-x_1)^2 + y^2 + z^2$

$$\rho_2^2 = (x-x_2)^2 + y^2 + z^2$$

$$2\rho_1 \frac{\partial \rho_1}{\partial x} = 2(x-x_1), \quad 2\rho_2 \frac{\partial \rho_2}{\partial x} = 2(x-x_2).$$

Now $\displaystyle \frac{\partial U}{\partial x} = \frac{1}{2}\cdot 2x + (1-m)(-1)\rho_1^{-2}\frac{\partial \rho_1}{\partial x} + m(-1)\rho_2^{-2}\frac{\partial \rho_2}{\partial x}$

$$= x - \frac{1-m}{\rho_1^2}\frac{\partial \rho_1}{\partial x} - \frac{m}{\rho_2^2}\frac{\partial \rho_2}{\partial x}.$$

$$= x - \frac{1-m}{\rho_1^2}\frac{x-x_1}{\rho_1} - \frac{m}{\rho_2^2}\frac{x-x_2}{\rho_2}.$$

$$= x - \frac{(1-m)}{\rho_1^3}(x-x_1) - \frac{m(x-x_2)}{\rho_2^3}$$

$$\frac{\partial U}{\partial y} = y - \frac{1-m}{\rho_1^3}y - \frac{m}{\rho_2^3}y.$$

$$\frac{\partial U}{\partial z} = -\frac{1-m}{\rho_1^3}z - \frac{m}{\rho_2^3}z.$$

eq^n 6, 7 & 8 reduces to

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x} \rightarrow (10)$$

$$\ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y} \rightarrow (11)$$

$$\ddot{z} = \frac{\partial U}{\partial z} \rightarrow (12)$$

$(10)\times 2\dot{x} + (11)\times 2\dot{y} + (12)\times 2\dot{z} \Rightarrow 2\dot{x}\ddot{x} + 2\dot{y}\ddot{y} + 2\dot{z}\ddot{z}$

$$= 2\dot{x}\frac{\partial U}{\partial x} + 2\dot{y}\frac{\partial U}{\partial y} + 2\dot{z}\frac{\partial U}{\partial z}.$$

$$\Rightarrow \frac{d}{dt}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = 2\frac{dU}{dt}$$

$$\Rightarrow \frac{d}{dt}v^2 = 2\frac{dU}{dt}.$$

$$\Rightarrow \frac{d}{dt}(v^2 - 2U) = 0$$

$$\Rightarrow v^2 - 2U = -c \Rightarrow v^2 = 2U - c \rightarrow (13)$$

where v is the speed of the infinitesimal mass is

~~an integral of the eqn of motion. Hence to prove~~

The eqn (13) involves one arbitrary constant. Therefore (13) is an integral of the eqn of motion. Hence to prove the complete soln of the eqn of motion, another five integrals are to be obtained. By further restricting the motion of the infinitesimal mass to the xy-plane, it is possible to reduce the no of consts required to three. Jacobi has shown that two of these are related to the third.

Therefore for a complete soln, there remaining to be found one new integral. Bruns has demonstrated that no new algebric integrals in rectangular co-ordinates exist.

The eqn (13) shows that v is a fn of (x,y). The constant c depends upon the initial position and velocity of the particle.

The curves of zero speed are given by $v=0$,

$$2U - c = 0$$

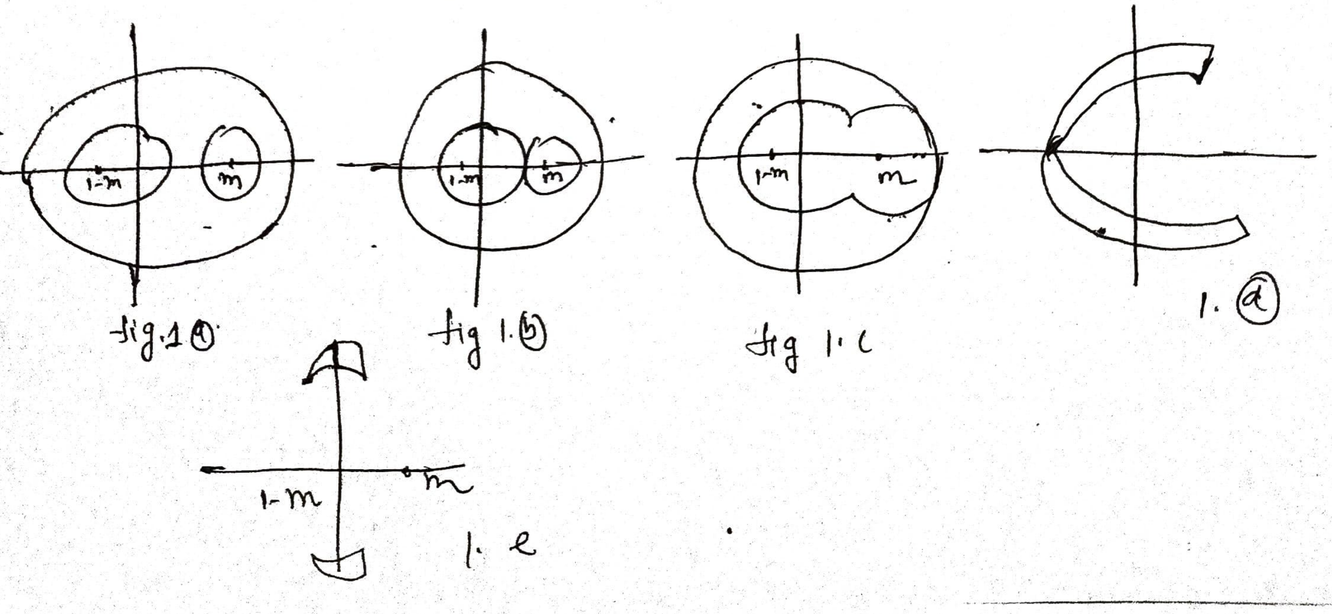
$$\Rightarrow x^2 + y^2 + \frac{2(1-m)}{\sqrt{(x-x_1)^2+y^2}} + \frac{2m}{\sqrt{(x-x_2)^2+y^2}} = C \rightarrow (14)$$

Motion of the particle occur only in those regions of the xy-plane for which $v^2 > 0$.

$$\Rightarrow 2U - C > 0$$

If c is large, (it must of course, be positive), we have three alternatives. $x^2+y^2 = c$ (nearly) or ρ_1 is very small or ρ_2 is very small

Hence, we have, roughly a cylinder with circular cross-section parallel to the z-axis and two small ovoids surrounding the finite masses. The larger ovoids is around the heavier mass $(1-m)$. for this values of c, the motion of the infinitesimal body can take place outside the cylinder or inside one of the ovoids. This situation in the xy plane is illustrated in figure 1.a



Now let c decrease, the 'cylinder' shrinks, and the ovoids expand untill they coalesce, this will take place in the xy-plane at a point close to m than to (1-m). This is illustrated in fig 1.(b) As C decreases further the wall of the cylinder meets the smaller and later the larger the ovoids (fig 1© & 1.ⓓ). firstly, we are left with two tad pole-like shapes that eventually think to point (fig 1.e) The parts of the figures 1 can be combined into one & this is in figure 2.