# ERRORS IN PHYSICS AN INTRODUCTION

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## WHY ERRORS ????????

Importance of it

# Definition of errors-

$\Delta$ = True Value − Measured value

# Types of Errors:

- Systematic Errors.
- Random Errors.

# Systematic Errors: Brief discussion-

- Origins of Systematic Errors-
  - a)instrumental errors.
  - b)personal errors.
  - c)external errors.

- How to minimize it-

# Random Errors: Brief Discussion-

- Error of the type which are beyond the control of the observer under the given conditions of a measurements, are known as random errors.
- General representation : $a \pm \Delta a$

- Relative error

(a) $\text{Relative error} = \dfrac{\text{Absolute error}}{\text{Actual value}}$

(b) $\text{Relative error} = \dfrac{\text{Maximum absolute error}}{\text{Measured value}}$

- Percentage error

Percentage error = Relative error × 100%

## Addition & Subtraction of errors:-

- Let $X = a \pm \Delta a$ and $Y = b \pm \Delta b$, then

  $X + Y = (a \pm \Delta a) + (b \pm \Delta b)$

  $= (a + b) \pm (\Delta a + \Delta b)$

  $= Z \pm \Delta z$

  where $Z = (a+b)$ & $\Delta z = (\Delta a + \Delta b)$

- Let $X = a \pm \Delta a$ and $Y = b \pm \Delta b$, then

  $X - Y = (a \pm \Delta a) - (b \pm \Delta b)$

  $= (a - b) \pm (\Delta a + \Delta b)$

  $= Z \pm \Delta z$

  where $Z = (a - b)$ & $\Delta z = (\Delta a + \Delta b)$

# Round-Off Error:-

- The process of dropping unwanted digits is known as rounding-off .
- The digits used to represent a number is called significant digits.

- Numbers are rounded-off according to the following rule:

To round-off a number to n significant digits, discard all digits to the right of the nth digit and if this discarded number is

1. less than 5 in (n + 1)th place, leave the nth digit unaltered. e.g., 7.893 to 7.89.

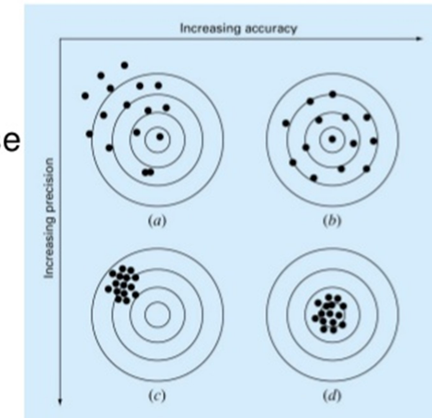
2. greater than 5 in (n + 1)th place, increase the nth digit by unity, e.g., 6.3456 to 6.346,

3. exactly 5 in (n + 1)th place, increase the nth digit by unity if it is odd, otherwise leave it unchanged. e.g., 12.675 ~ 12.68 and 12.685 ~ 12.68.

The number thus rounded-off is said to be correct to n significant figures. The error introduced by rounding-off numbers to given decimal places is known as round-off error

# Accuracy and Precision

- Accuracy how close your solution is
- Precision is how close your repetition of the solution are!



# Taylor's Expansion

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + f''(x_0)\frac{(x - x_0)^2}{2!} + \ldots$$

$$f(x) = f(x_0) + f'(x_0)h + f''(x_0)\frac{h^2}{2!} + O(h^3)$$

## 10.5 ERROR DUE TO APPROXIMATION OF THE FUNCTION

Let $z = f(x, y)$ be a function of two variables $x$ and $y$.

If $\delta x$, $\delta y$ be the errors in $x$ and $y$, then the error in $z$ is given by $z + \delta z = f(x + \delta x, y + \delta y)$.

Expanding $f(x, y)$ by Taylor's series, we get

$$z + \delta z = f(x, y) + \left(\frac{\partial f}{\partial x}\delta x + \frac{\partial f}{\partial y}\delta y\right) + \text{terms involving higher powers of } \delta x \text{ and } \delta y. \quad ...(1)$$

If $\delta x$ and $\delta y$ be so small that their squares and higher powers can be neglected, then (1) can be written as

$$\delta z = \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y \quad \text{(app.)}$$

In general, if $z = f(x_1, x_2, ..... x_n)$ and there are errors in $x_1, x_2 ..... x_n$, then

$$\delta z = \frac{\partial z}{\partial x_1}dx_1 + \frac{\partial z}{\partial x_2}dx_2 + \frac{\partial z}{\partial x_3}dx_3 + ..... + \frac{\partial z}{\partial x_n}dx_n.$$

**Example 2.** *If* $u = \frac{5x^3y^4}{z^5}$ *and errors in x, y, z be 0.001, and compute the relative maximum error when x = 1, y = 1, z = 1.*

**Solution.**

$$u = \frac{5x^3y^4}{z^5} \quad \text{...(1)}$$

$$\delta x = \delta y = \delta z = 0.001$$

and

$$x = y = z = 1$$

Differentiating (1) partially, with respect to '$x$', we get

$$\frac{\delta u}{\delta x} = \frac{15x^2y^4}{z^5}, \quad \frac{\delta u}{\delta y} = \frac{20x^3y^3}{z^5}, \quad \frac{\delta u}{\delta z} = -\frac{25x^3y^4}{z^6}$$

Now, we know that

$$\delta u = \frac{\partial u}{\partial x}\delta x + \frac{\partial u}{\partial y}\delta y + \frac{\partial u}{\partial z}\delta z$$

$$= \frac{15x^2y^4}{z^5}\delta x + \frac{20x^3y^3}{z^5}\delta y - \frac{25x^3y^4}{z^6}\delta z$$

The error being maximum

$$(\delta u)_{max} = \left|\frac{15x^2y^4}{z^5}\delta x\right| + \left|\frac{20x^3y^3}{z^5}\delta y\right| + \left|\frac{25x^3y^4}{z^6}\delta z\right|$$

$$= \left|\frac{15(1)(1)}{(1)}(0.001)\right| + \left|\frac{20(1)(1)}{(1)}(0.001)\right| + \left|\frac{25(1)(1)}{(1)}(0.001)\right|$$

$$= 0.015 + 0.020 + 0.025 = 0.06$$

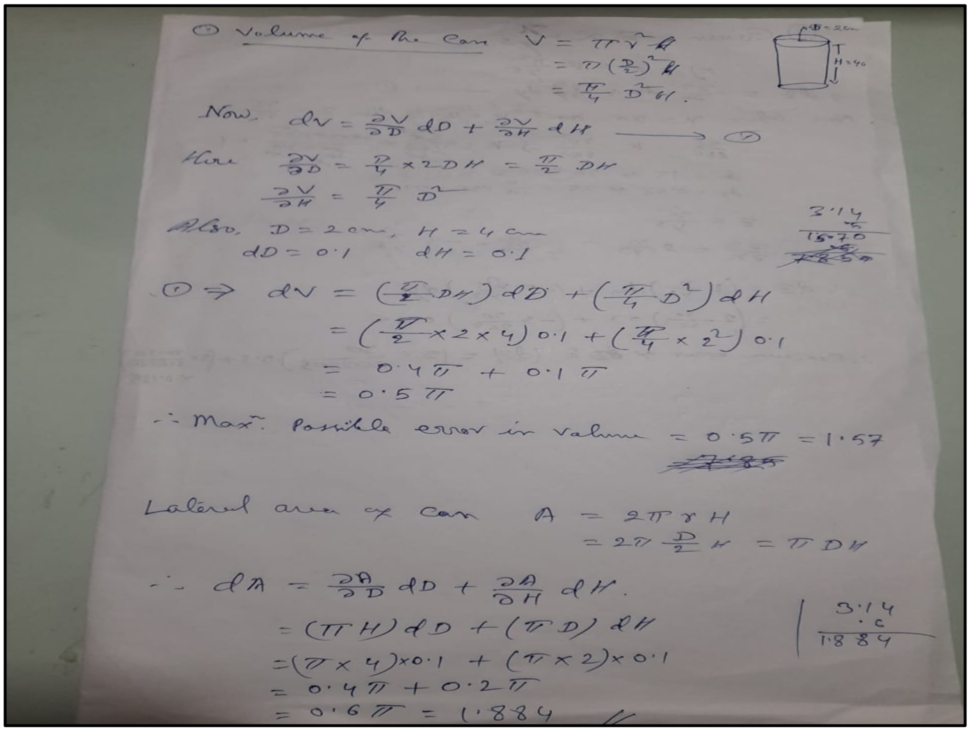
$$\text{Relative error} = \frac{(\delta u)_{max}}{u} = \frac{0.06}{5} = 0.012 \quad \text{Ans.}$$

## Home Work Exercise

(1) The diameter and altitude of a can in the shape of a rigid circular cylinder are measured as 2 cm & 4 cm respectively. The possible error in each measurement is 0.1 cm. Find approximately the maximum possible error in the values computed for the volume and lateral (except base and top surfaces) surface area.

(2) If z varies directly as the square of x and inversely as the cube of y and the possible errors in measuring x and y are 1% and 0.5% respectively, Find the amount of error in z. Give that, when x and y are 30 and 25 respectively $z = 36/625$

(3) You measure the radius of a circle and find $(4.9 \pm 0.1)$ cm Express the area with error (KNU-2019)



(i) Volume of the Can: $V = \pi r^2 H$

$= \pi \left(\frac{D}{2}\right)^2 H$

$= \frac{\pi}{4} D^2 H.$



Now, $dV = \frac{\partial V}{\partial D} dD + \frac{\partial V}{\partial H} dH \longrightarrow (1)$

Here $\frac{\partial V}{\partial D} = \frac{\pi}{4} \times 2DH = \frac{\pi}{2} DH$

$\frac{\partial V}{\partial H} = \frac{\pi}{4} D^2$

Also, $D = 2$ cm, $H = 4$ cm

$dD = 0.1 \quad dH = 0.1$

$(1) \Rightarrow dV = \left(\frac{\pi}{2} DH\right) dD + \left(\frac{\pi}{4} D^2\right) dH$

$= \left(\frac{\pi}{2} \times 2 \times 4\right) 0.1 + \left(\frac{\pi}{4} \times 2^2\right) 0.1$

$= 0.4\pi + 0.1\pi$

$= 0.5\pi$

$\therefore$ Max. possible error in volume $= 0.5\pi = 1.57$

Lateral area of can $A = 2\pi r H$

$= 2\pi \frac{D}{2} H = \pi D H$

$\therefore dA = \frac{\partial A}{\partial D} dD + \frac{\partial A}{\partial H} dH.$

$= (\pi H) dD + (\pi D) dH$

$= (\pi \times 4) \times 0.1 + (\pi \times 2) \times 0.1$

$= 0.4\pi + 0.2\pi$

$= 0.6\pi = 1.884$

$3.14 \times .6 = 1.884$

(2) Given $z \propto \frac{x^2}{y^3}$

$\Rightarrow z = k\frac{x^2}{y^3}$

$\Rightarrow dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy \longrightarrow (1)$

Now, when $x = 30$ and $y = 25$ then $z = \frac{36}{625}$

$\therefore \frac{36}{625} = k\frac{30^2}{25^3} \Rightarrow k = \frac{36}{625} \times \frac{25 \times 25 \times 25}{30 \times 30} = 1$

$dx = 1\%$ of $30 = 0.3$

$dy = 0.5\%$ of $25 = \frac{.5}{100} \times 25 = 0.125$

$z = \frac{x^2}{y^3}$

$\frac{\partial z}{\partial x} = 2\frac{x}{y^3}$, $\frac{\partial z}{\partial y} = -3\frac{x^2}{y^4}$

$\therefore dz = \left(2\frac{x}{y^3}\right)dx + \left(-3\frac{x^2}{y^4}\right)dy$

$= \left(2\frac{30}{25^3}\right)0.3 + \left(-3\frac{30^2}{25^4}\right)0.125$

$\therefore$ Maximum error of z is $|dz| = \left(2 \times \frac{30}{25 \times 25 \times 25}\right)0.3 + \left(3 \times \frac{30 \times 30}{625 \times 625}\right) \times 0.125$

$= \ldots$

3) Radius $r = (4.9 \pm 0.1)$ cm

i.e. $r = 4.9$ , $dr = 0.1$

Area $A = \pi r^2 = 3.14 \times (4.9)^2$

$dA = \left(\frac{\partial A}{\partial r}\right) dr$

$= \pi \, 2r \times dr$

$= 3.14 \times 2 \times 4.9 \times 0.1$

$\therefore$ Area $= A \pm dA$.

$= \left[3.14 \times (4.9)^2 \pm 3.14 \times 2 \times 4.9 \times 0.1\right]$ cm$^2$

$= ( \quad \pm \ldots)$ cm$^2$

# THANK YOU

To Error is Human