ELECTRIC DIPOLE

ELECTRIC FIELD AND POTENTIAL- II

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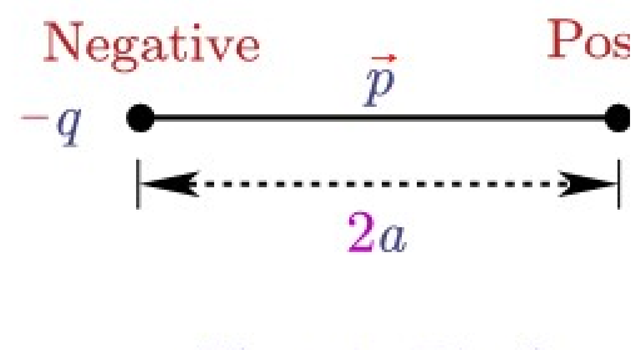
Electric Dipole

An **electric dipole** is defined as a couple of opposite charges q and –q separated by a distance d.

Electric dipole moment:- The dipole moment of an electric field is a vector whose magnitude is charge times the separation between two opposite charges.

Magnitude of **dipole moment** is

$$|\vec{p}| = q \times 2a.$$

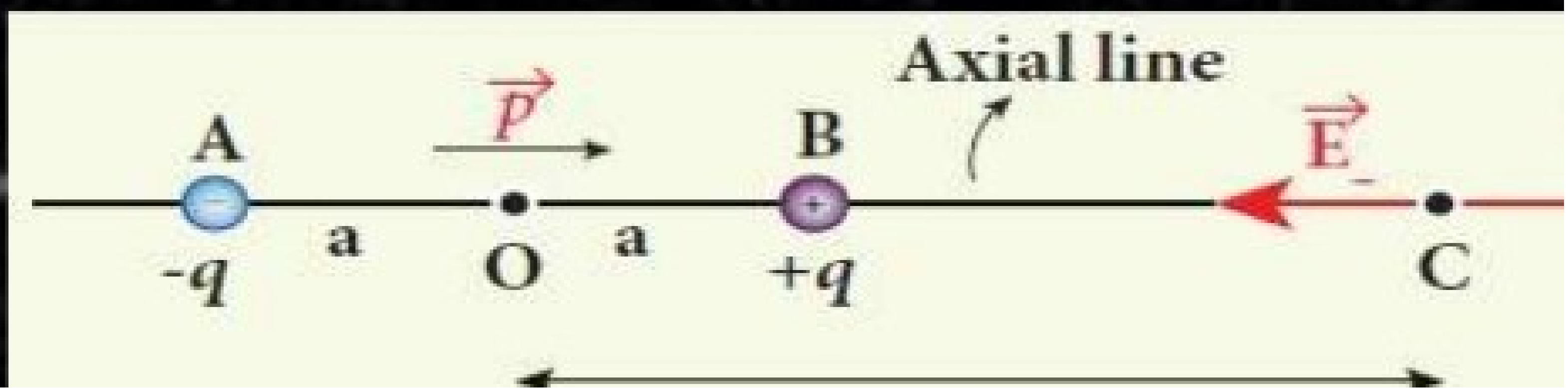


The Electric dipole moment direction is from negative charge to positive charge.

The SI **unit** of **dipole moment** is Coulomb-meter(C−m)

Electric field due to an electric d
at points on the axial line

- Consider an electric dipole placed on th
  axis
- A point C is located at a distance of *r fr*
  midpoint O of the dipole along the axia



The electric field at a point C due tc

$$\vec{E}_{+} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} \text{ along BC}$$

Since the electric dipole moment ve is from −q to +q and is directed alo the above equation is rewritten as

$$\vec{E}_{+} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p}$$

where $\hat{p}$ is the electric dipole mome

- The electric field at a point C due to

$$\vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p}$$

Since +q is located closer to the point C
−q, $\vec{E}_+$ is stronger than $\vec{E}_-$. Therefor
length of the $\vec{E}_+$ vector is drawn larger

- The total electric field at point calculated using the superp principle of the electric field

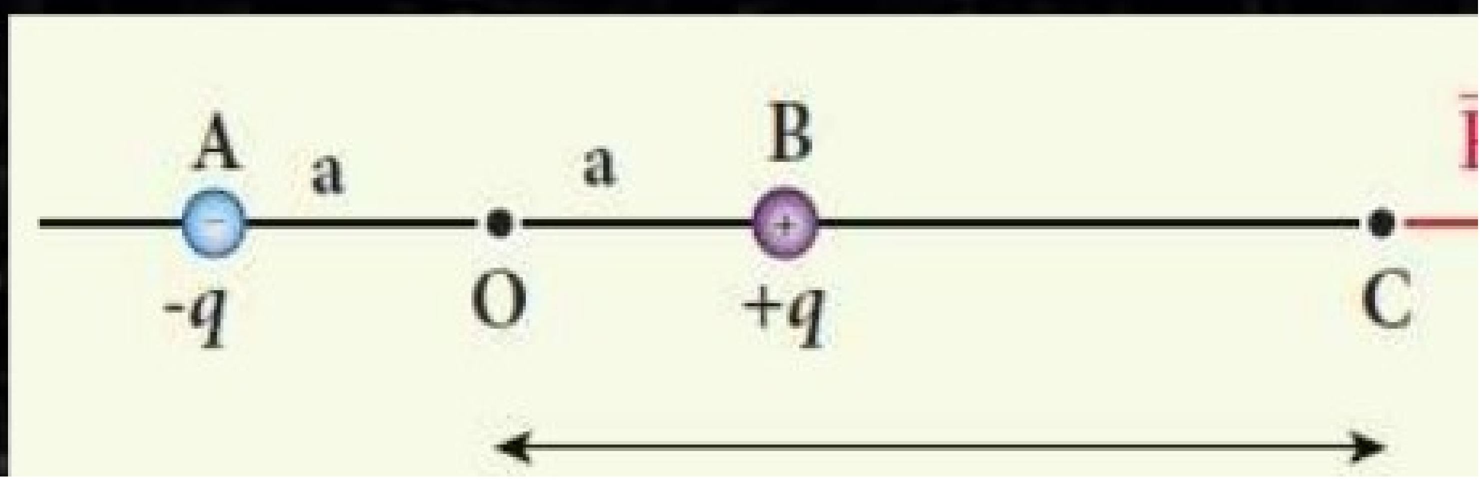
$$\vec{E}_{tot} = \vec{E}_{+} + \vec{E}_{-}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a}$$

$$\vec{E} = \frac{q}{} \left(\frac{1}{} - \frac{1}{} \right.$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0} q \left(\frac{4ra}{(r^2 - a^2)^2} \right) \hat{p}$$

Note that the total electric field is alon
since $+q$ is closer to C than –q.



If the point C is very far away fr
dipole then (r >> a). Under this li
term $(r^2 - a^2)^2 \approx r^4$.

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4aq}{r^3}\right)\hat{p} \quad (r >> a)$$

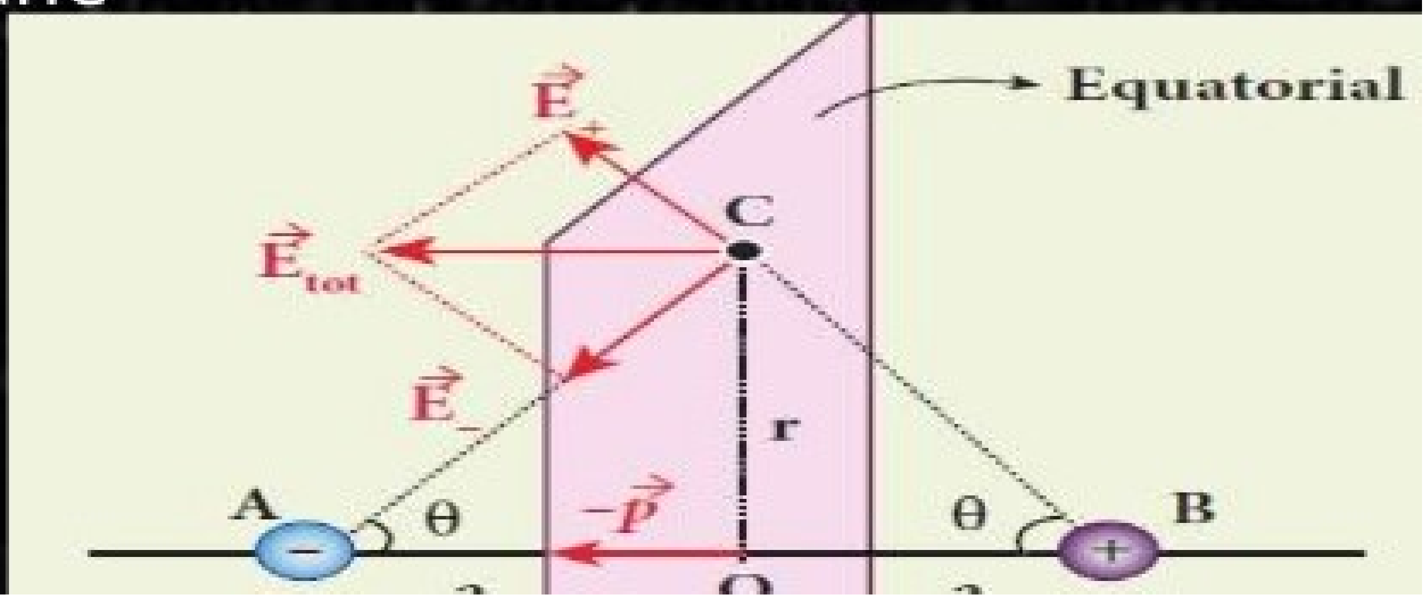
since $2aq\,\hat{p}$

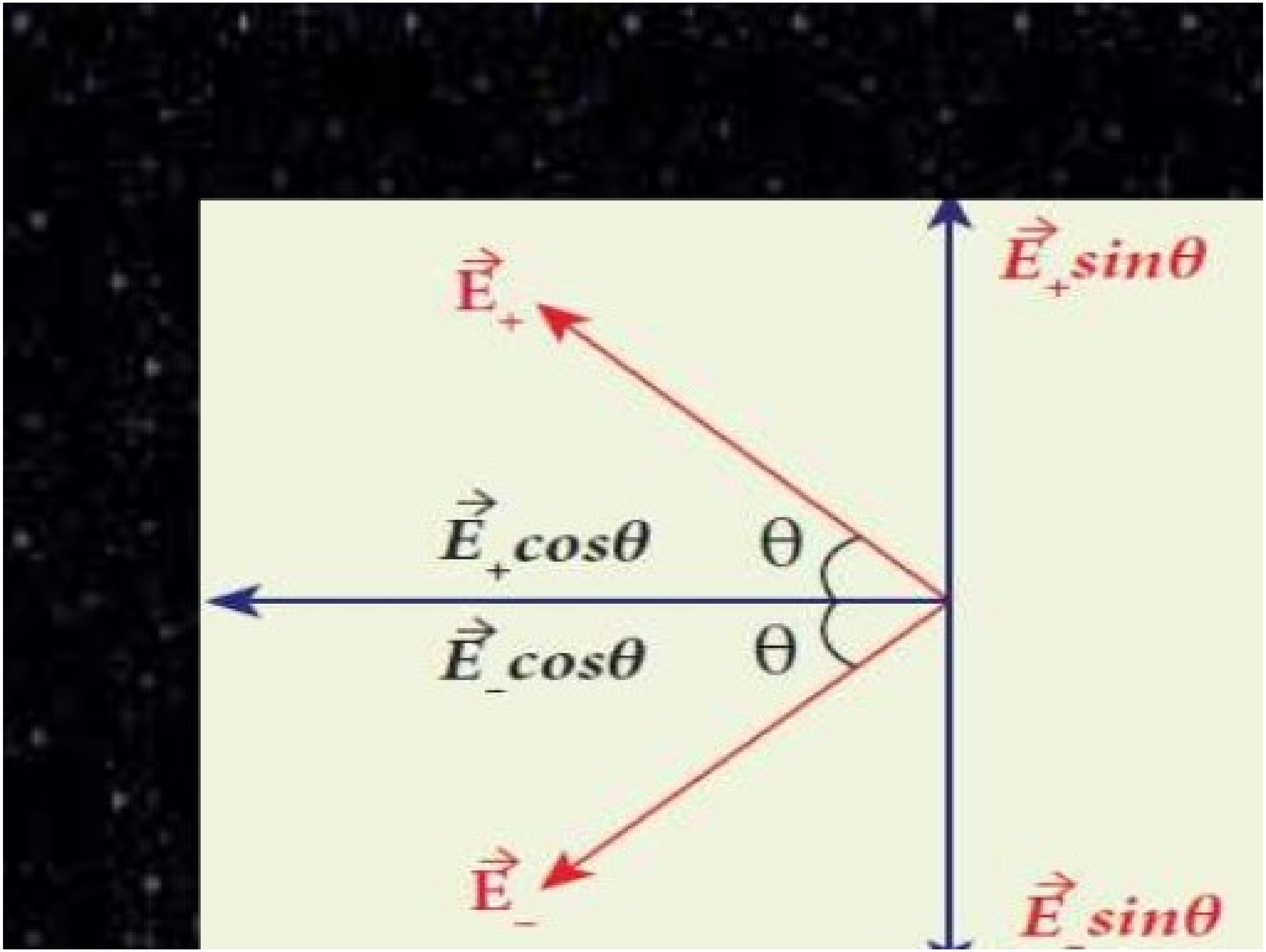
$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3} \quad (r >$$

If the point C is chosen on the left s

Electric field due to an electric d
at a point on the equatorial pl

- Consider a point C at a distance r fr midpoint O of the dipole on the eq plane




$\vec{E}_+ sin\theta$
$\vec{E}_+$
$\vec{E}_+ cos\theta$
θ
θ
$\vec{E}_- cos\theta$
$\vec{E}_-$
$\vec{E}_- sin\theta$

Since the point C is equi-distant fr
and –q, the magnitude of the electric
of +q and –q are the same.

The direct
$\vec{E}_+$ is along BC and the direction o
along CA. $\vec{E}_+$ and $\vec{E}_-$ are resolved int
components; one component parallel
dipole axis and the other perpendicu
it.

The perpendicular components $|\vec{E}$

The magnitude total electric field at point C is the the parallel components of $\vec{E}_+$ and its direction is along $-\hat{p}$

$$\vec{E}_{tot} = -\left|\vec{E}_+\right|\cos\theta\,\hat{p} - \left|\vec{E}_-\right|\cos$$

The magnitudes $\vec{E}_+$ and $\vec{E}_-$ are the san are given by

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0}\frac{2q\cos\theta}{\left(r^2+a^2\right)}\hat{p}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{2qa}{\left(r^2+a^2\right)^{\frac{3}{2}}}\hat{p}$$

$$\text{since } \cos\theta = \frac{a}{\sqrt{\ldots}}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_o}\frac{\vec{p}}{\left(r^2+a^2\right)^{\frac{3}{2}}}$$

since $\vec{p} = 2qa\hat{p}$

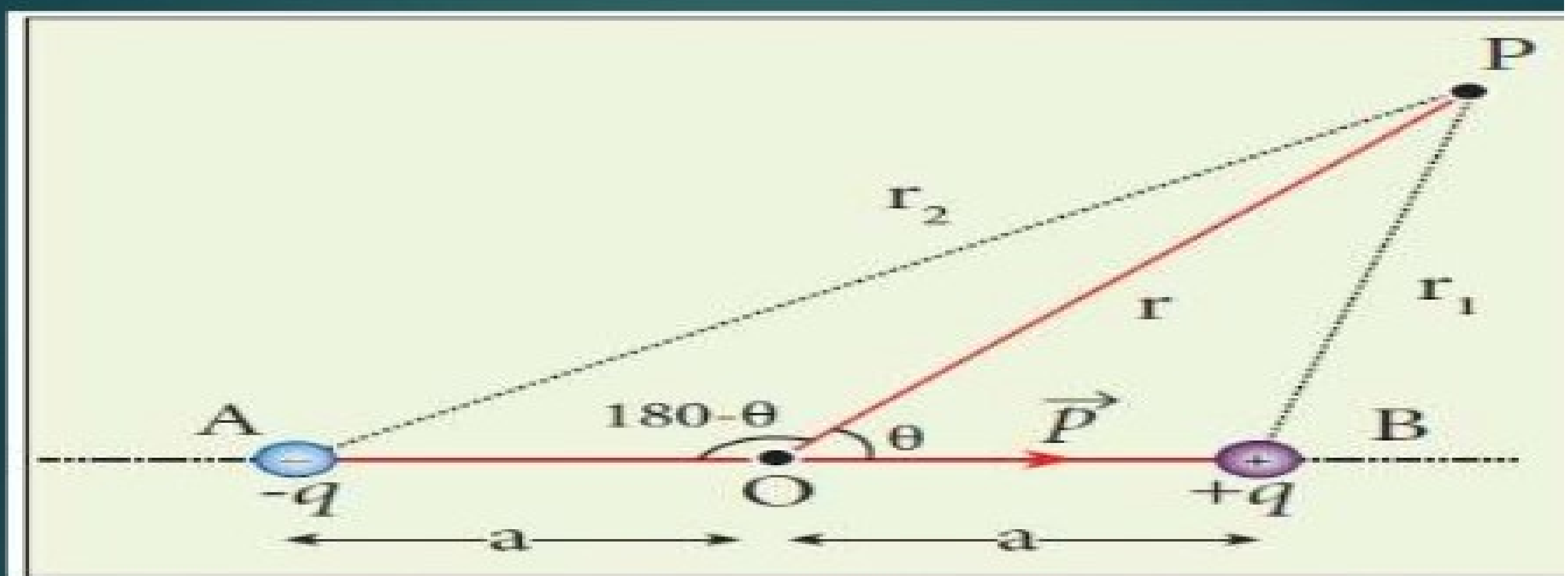
- At very large distances (r>>a),

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_o}\frac{\vec{p}}{r^3} \qquad (r>>a)$$

- It is inferred that for very large distan magnitude of the electric field at point dipole axis is twice the magnitude electric field at points on the equatoria
- The direction of the electric field at p the dipole axis is directed along the d of dipole moment vector p but at p the equatorial plane it is directed opp the dipole moment vector, that is alon

Electrostatic potential at a point due to an electric dipole

- Consider two equal and opposite charge separated by a small distance 2a
- The point P is located at a distance r from th midpoint of the dipole.



Let r_1 be the distance of point P from +q an
r_2 be the distance of point P from −q.

Potential at P due to charge +q $= \frac{1}{4\pi\varepsilon_o}\frac{q}{r_1}$

Potential at P due to charge −q $= -\frac{1}{4\pi\varepsilon_o}\frac{q}{r_2}$

Total potential at the point P,

- Suppose if the point P is far away from the dipole, such that r>>a, then equation can be expressed in terms of r.
- By the cosine law for triangle BOP

$$r_1^2 = r^2 + a^2 - 2ra\cos\theta$$

- Since the point P is very far from dipole, then r>>a. As a result the term a^2/r^2 is very small and can be neglected. Therefore

$$r_1^2 = r^2\left(1 - 2a\frac{\cos\theta}{r}\right)$$

$$\text{(or)} \quad r_1 = r\left(1 - \frac{2a}{r}\cos\theta\right)^{\frac{1}{2}}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}$$

Since $\frac{a}{r} << 1$, we can use binomial theorem and retain the terms up to first order

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right)$$

Similarly applying the cosine law for triangl
AOP,

$$r_2^2 = r^2 + a^2 - 2ra\cos(180 - \theta)$$

since $\cos(180 - \theta) = -\cos\theta$ we get

$$r_2^2 = r^2 + a^2 + 2ra\cos\theta$$

Neglecting the term $\frac{a^2}{r^2}$ (because r>>a)

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Neglecting the term $\frac{a^2}{r^2}$ (because r>>a)

$$V = \frac{1}{4\pi\varepsilon_{o}} q \left(\frac{1}{r}\left(1 + a\frac{\cos\theta}{r}\right) - \frac{1}{r}\left(1 - a\frac{\cos\theta}{r}\right) \right)$$

$$V = \frac{q}{4\pi\varepsilon_{o}} \left(\frac{1}{r}\left(1 + a\frac{\cos\theta}{r} - 1 + a\frac{\cos\theta}{r}\right) \right)$$

$$V = \frac{1}{\quad}\frac{2aq}{\quad}\cos\theta$$

But the electric dipole moment $p = 2qa$ and we get,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

Now we can write p cosθ = $\vec{p}\cdot\hat{r}$, where $\hat{r}$ is the unit vector from the point O to point P. Hence the electric potential at a point P due to an electric dipole is given by

Special cases

- **1.** If the point P lies on the axial line of the dipole on t of +q, then θ = 0. Then the electric potential becomes

$$V = \frac{1}{4\pi\varepsilon_o}\frac{p}{r^2}$$

- **2.** If the point P lies on the axial line of the dipole on t of –q, then θ = 180°, then

$$V = -\frac{1}{4\pi\varepsilon_o}\frac{p}{r^2}$$

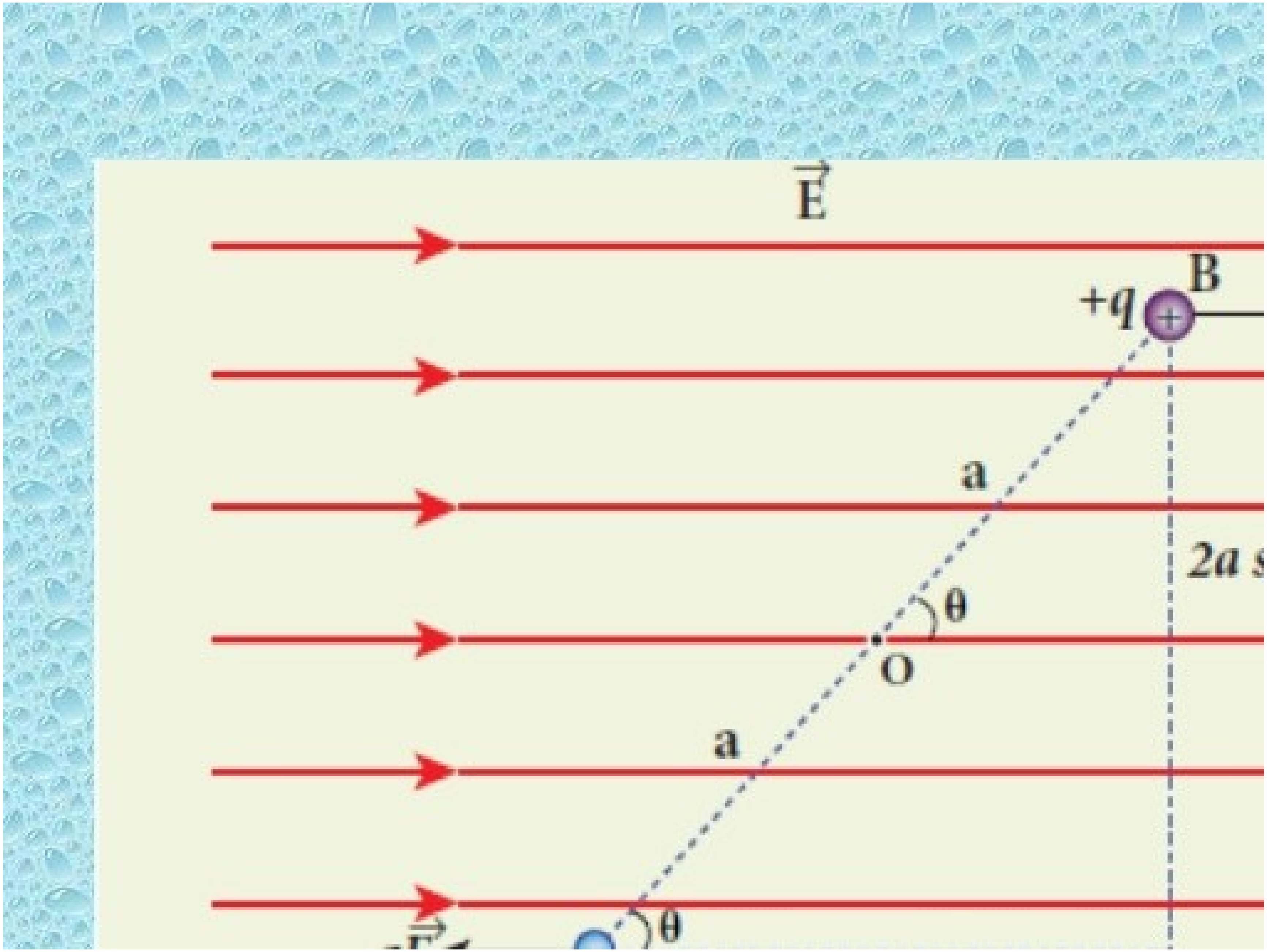
- 3.If the point P lies on the equatorial line of the dipol = 90o. Hence

$$V = 0$$

- The potential due to an electric dipole falls a $1/r^2$ and the potential due to a single poir charge falls as $1/r$.
- Thus the potential due to the dipole falls faste than that due to a monopole (point charge).
- As the distance increases from electric dipole the effects of positive and negative charge

Torque experienced by an electric dipole in the uniform electric field

- Consider an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field E whose field lines are equally spaced and point in the same direction
- The charge +q will experience a force q Ein the direction of the field and charge –q will experience a force –qE in a direction opposite to the field.


E
B
+q
a
2a s
θ
O
a
θ

- The total torque on the dipole about the point O

$$\vec{\tau} = \overrightarrow{OA} \times \left(-q\vec{E}\right) + \overrightarrow{OB} \times q\vec{E}$$

- The torque is perpendicular to the plane of the paper and is directed into it.
- The magnitude of the total torque

$$\vec{\tau} = \left|\overrightarrow{OA}\right| \left|\left(-q\vec{E}\right)\right| \sin\theta + \left|\overrightarrow{OB}\right| \left|q\vec{E}\right| \sin\theta$$

$$\tau = qE \cdot 2a\sin\theta$$

- where θ is the angle made by p with E *. Since p = 2aq, the torque is written in terms of the vector product as*

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The magnitude of this torque is $\tau = pE\sin\theta$ and is maximum when $\theta = 90^\circ$.

This torque tends to rotate the dipole and align it with the electric field $\vec{E}$. Once $\vec{p}$ is aligned with $\vec{E}$, the total torque on the dipole becomes zero.

Thanks