Moments

In statistics, moments are used to describe the various characteristics of a frequency distribution like central tendency, variation, skewness and kurtosis.

It can be seen that the formula for a moment co-efficient is identical with that for an arithmatic mean. This identity led statisticians to speak of the arithmatic mean as the "first moment about the origin".

The statistical definition of the term moment is:

Let x be used to represent the deviation of any item in a distribution from the arithmatic mean of that distribution. The arithmatic mean of the various powers of these deviations is called the moment of the distribution.

If we take the mean of the first power of the deviations, we get the first moment about the mean; the mean of the squares of the deviations gives us the second moment and so on. The moments about mean called the 'central moment' are denoted by Greek letter μ. Thus μ_1 stands for first moment about mean, μ_2 stands for second moment about mean etc.

Symbolically,

$$\mu_1 = \frac{\Sigma(X-\bar{X})}{N} \text{ or } \frac{\Sigma x}{N}$$

(Since sum of the deviation of items from arithmatic mean is always zero, μ_1 would always be zero)

$$\mu_2 = \frac{\Sigma(X-\bar{X})^2}{N} \text{ or } \frac{\Sigma x^2}{N}$$

$$\mu_3 = \frac{\Sigma(X-\bar{X})^3}{N} \text{ or } \frac{\Sigma x^3}{N}$$

For frequency distribution

$$\mu_1 = \frac{\Sigma f(X-X)}{N} \text{ or } \frac{\Sigma f x}{N}$$

$$\mu_2 = \frac{\Sigma f(X-\bar{X})^2}{N} \text{ or } \frac{\Sigma f x^2}{N}$$

$$\mu_3 = \frac{\Sigma f(X-\bar{X})^3}{N} \text{ or } \frac{\Sigma f x^3}{N}$$

$$\mu_4 = \frac{\Sigma f(X-\bar{X})^4}{N} \text{ or } \frac{\Sigma f x^4}{N}$$

Moments can be extended to higher powers in a similar way, but generally in practice the first four moments suffice.

Two important constants of a distribution are calculated from μ_2, μ_3 and μ_4. They are:

(i) $$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

(ii) $$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

β_1 measures skewness and β_2 kurtosis.

In a symmetrical distribution all odd moments ie μ_1, μ_3 etc would always be zero. The reason is that if the curve is symmetrical there will be a deviation below the mean which exactly equals each deviation above the mean and, therefore, positive deviations and negative deviations will exactly balance out and when added will cancel out, i.e $\Sigma(x-\bar{x})$ would always be zero. Of course, if the deviations are raised to even powers, their sign will always be positive and they will no longer cancel out. But the sum of the odd powers will all be equal to zero on account of the cancellations. Thus odd moments are always zero in symmetrical distribution. However, this rule does not hold true in asymmetrical distribution.