Non homogeneous Differential Equation

Ex① $(x+y+1)dx = (2x+2y+1)dy$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y+1}{2x+2y+1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x+y)+1}{2(x+y)+1}$$

Let, $v = x+y$

$\therefore \frac{dv}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow \frac{dv}{dx} - 1 = \frac{dy}{dx}$

$$\Rightarrow \frac{dv}{dx} - 1 = \frac{v+1}{2v+1}$$

$$\Rightarrow \frac{dv}{dx} = 1 + \frac{v+1}{2v+1}$$

$$\Rightarrow \frac{dv}{dx} = \frac{2v+1+v+1}{2v+1}$$

$$\Rightarrow \frac{dv}{dx} = \frac{3v+2}{2v+1}$$

$$\Rightarrow \frac{2v+1}{3v+2}dv = dx$$

$$\Rightarrow \int \frac{(2v+1)dv}{3v+2} = \int dx$$

$3v+2 \overline{)\, 2v+1 \,(\, \frac{2}{3}}$

$2v + \frac{4}{3}$

$1 - \frac{4}{3} = -\frac{1}{3}$

$$\Rightarrow \int \left(\frac{2}{3} - \frac{\frac{1}{3}}{3v+2}\right)dv = \int dx$$

$$\Rightarrow \frac{2}{3}\int dv - \frac{1}{3}\int \frac{dv}{3v+2} = \int dx$$

$$\Rightarrow \frac{2}{3}v - \frac{1}{3}\cdot\frac{1}{3}\log(3v+2) = x + c$$

$$\Rightarrow \frac{2}{3}(x+y) - \frac{1}{9}\log\{3(x+y)+2\} = x + c$$

Ans

Ex (2)

$$(2x-2y+5)\,dy-(x-y+3)\,dx=0$$

$$\Rightarrow (2x-2y+5)\,dy=(x-y+3)\,dx$$

$$\Rightarrow \frac{dy}{dx}=\frac{x-y+3}{2x-2y+5}=\frac{(x-y)+3}{2(x-y)+5}$$

Let, $v=x-y$

$\therefore \frac{dv}{dx}=1-\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}=1-\frac{dv}{dx}$

$$\Rightarrow 1-\frac{dv}{dx}=\frac{v+3}{2v+5}$$

$$\Rightarrow \frac{dv}{dx}=1-\frac{v+3}{2v+5}$$

$$=\frac{2v+5-v-3}{2v+5}$$

$$=\frac{v+2}{2v+5}$$

$$\Rightarrow \frac{2v+5}{v+2}\,dv=dx$$

$$\Rightarrow \int\frac{(2v+5)\,dv}{v+2}=\int dx$$

$$\Rightarrow \int\left(2+\frac{1}{v+2}\right)dv=\int dx$$

$$\Rightarrow 2\int dv+\int\frac{dv}{v+2}=\int dx$$

$$\Rightarrow 2v+\log(v+2)=x+\log c$$

$$\Rightarrow 2(x-y)-x=\log c-\log(v+2)$$

$$\Rightarrow x-2y=\log\frac{c}{v+2}=\log\frac{c}{x-y+2}$$

$$\therefore \frac{c}{x-y+2}=e^{x-2y}\Rightarrow (x-y+2)e^{x-2y}=c$$

$v+2\overline{)\,2v+5\,(2}$

$2v+4$

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Ex (3) $\frac{dy}{dx} = \frac{6x-2y-7}{3x-y-6}$

$\Rightarrow \frac{dy}{dx} = \frac{2(3x-y)-7}{(3x-y)-6}$

Let $v = 3x - y$

$\therefore \frac{dv}{dx} = 3 \cdot 1 - \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = 3 - \frac{dv}{dx}$

$\Rightarrow 3 - \frac{dv}{dx} = \frac{2v-7}{v-6}$

$\Rightarrow \frac{dv}{dx} = 3 - \frac{2v-7}{v-6}$

$\Rightarrow \frac{dv}{dx} = \frac{(3v-18)-2v+7}{v-6} = \frac{v-11}{v-6}$

$\Rightarrow \frac{v-6}{v-11} dv = dx$

$\Rightarrow \int \frac{(v-6)}{v-11} dv = \int dx$

$v-11 \overline{)\, v-6\ (1}$
$\quad v-11$
$\quad -6+11 = 5$

$\Rightarrow \int \left(1 + \frac{5}{v-11}\right) dv = \int dx$

$\Rightarrow \int dv + 5 \int \frac{dv}{v-11} = \int dx$

$\Rightarrow v + 5 \log(v-11) = x + \log c$

$\Rightarrow v - x = \log c - \log (v-11)^5$

$\Rightarrow 3x - y - x = \log \frac{c}{(v-11)^5}$

$\Rightarrow 2x - y = \log \frac{c}{(3x-y-11)^5}$

$\Rightarrow \frac{c}{3x-y-11} = e^{2x-y}$

$\Rightarrow (3x-y-11) e^{2x-y} = c$ Ans