

ELECTRIC FIELD AND POTENTIAL

Part - I

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Charges

- ✦ All matter is made up of atoms and molecules;
 - ◆ contain charged particles,
 - ◆ the proton and electron.
- ✦ The charges on each are equal;
 - ◆ but opposite in sign.

Charges

- ✦ The quantity of charge is measured coulombs (C).
- ✦ $1\text{C} = \text{charge carried by } 6.25 \times 10^{18} \text{ e}$
- ✦ The charge on one electron = $-1.6 \times 10^{-19} \text{ C}$
- ✦ The charge on one proton = $+1.6 \times 10^{-19} \text{ C}$

Coulomb's Law

Coulomb's law:

$$F = k \frac{Q_1 Q_2}{r^2}, \quad [\text{magnitudes}]$$

This equation gives the magnitude of the force.

The proportionality constant k can also be written in terms of ϵ_0 , the permittivity of free space:

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2},$$
$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2.$$

The Electric Field

For a point charge:

$$E = \frac{F}{q} = \frac{kqQ/r^2}{q}$$

$$E = k \frac{Q}{r^2}; \quad \text{[single point charge]}$$

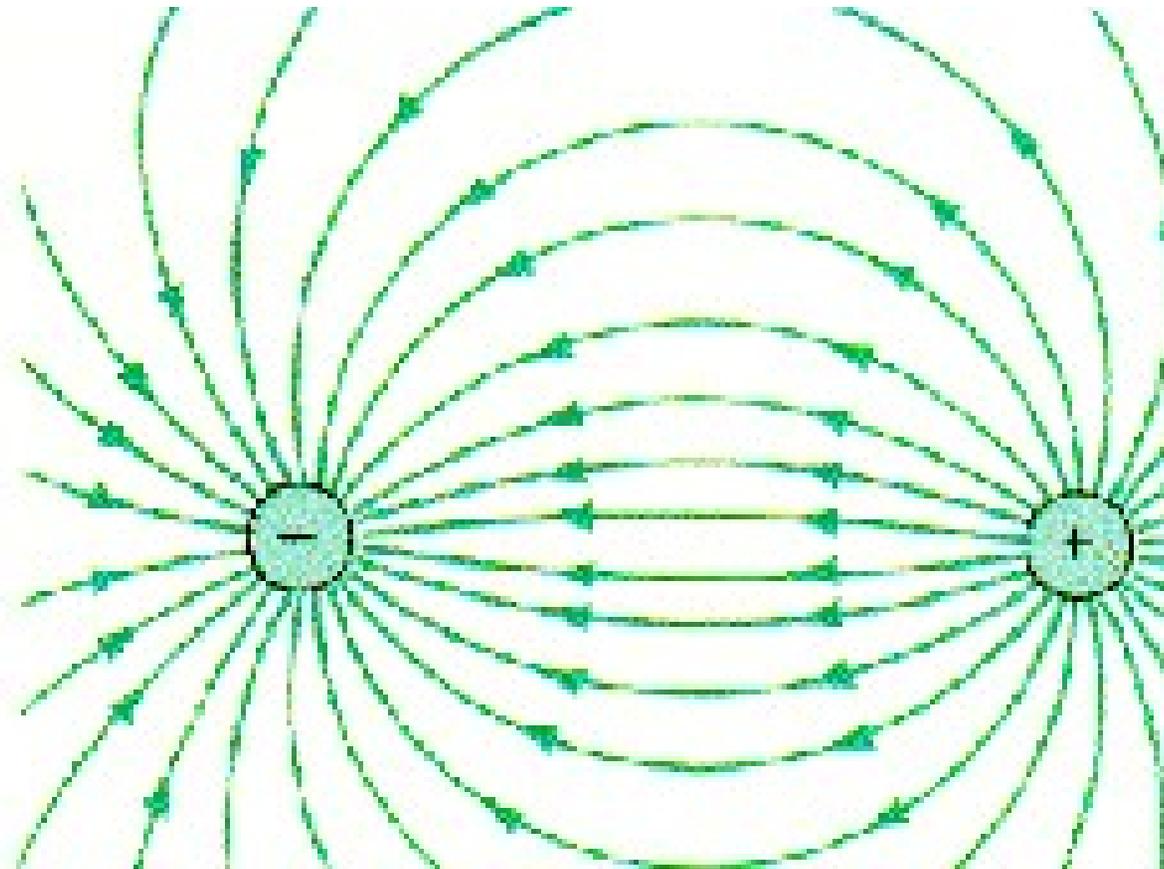
$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}. \quad \text{[single point charge]}$$

Electric Field Vector, E

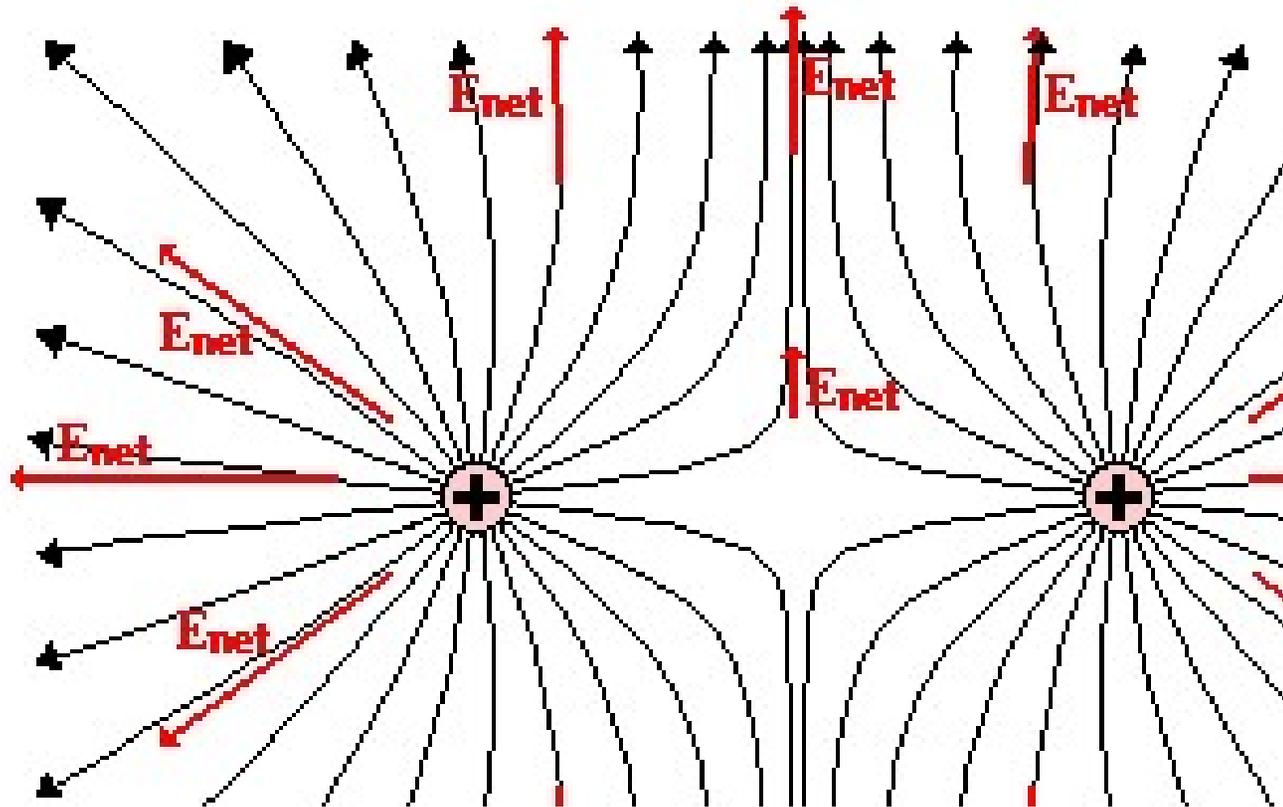
Electric Field is designed as follows –

- $E = F / q_0$ where q_0 , positive test charge
- **E is a vector quantity**
 - Direction indicated by small + test charge
- **Unit: N/C**

Electric Field Lines

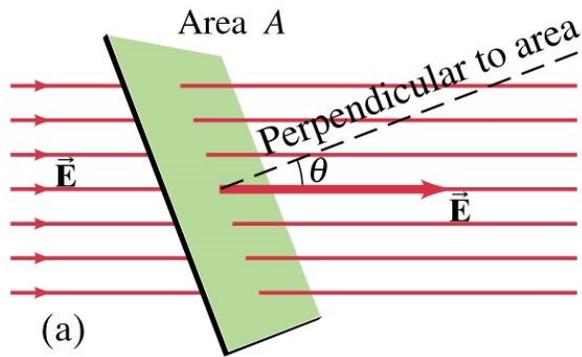


Electric Field Lines of two Positive Charges



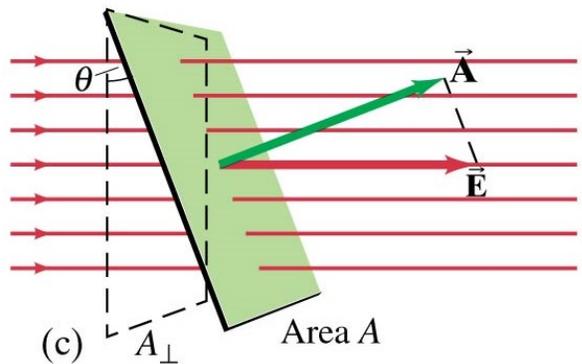
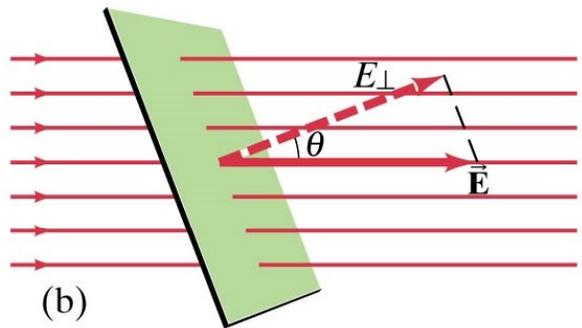
FLUX OF AN ELECTRIC FIELD

- ▶ Electric flux is the total number of lines of force passing through a surface.
- ▶ In physical sense, electric flux is defined as:
"The total number of lines of force passing through the unit area of a surface held *perpendicularly*."
- ▶ Mathematically the electric flux is defined as:
"The dot product of electric field intensity



Electric flux:

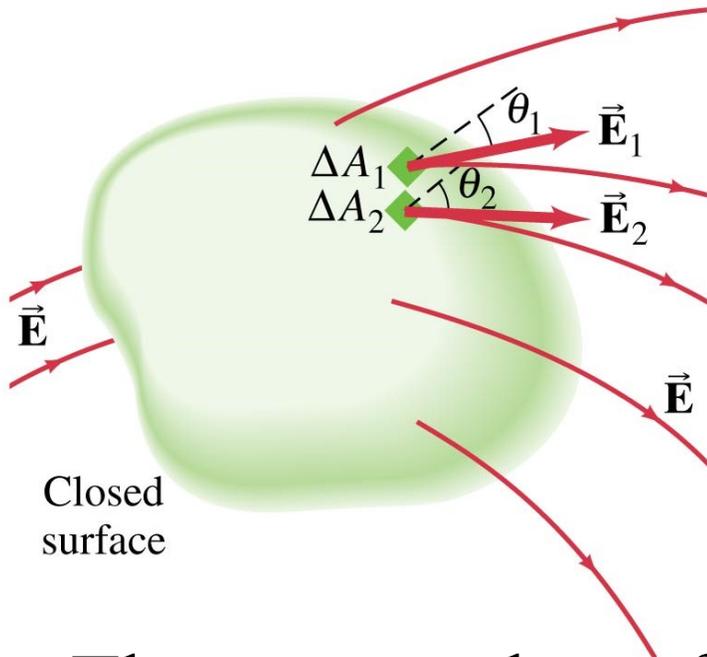
$$\Phi_E = E_{\perp} A = EA_{\perp} = EA \cos \theta,$$



* Electric flux through an area is proportional to the total number of field lines crossing the area.

Gauss's Law

Flux through a closed surface:



$$\begin{aligned}\Phi_E &= E_1 \Delta A_1 \cos \theta_1 + E_2 \Delta A_2 \cos \theta_2 + \dots \\ &= \sum E \Delta A \cos \theta = \sum E_{\perp} \Delta A,\end{aligned}$$

The net number of field lines through the surface is proportional to the charge enclosed, and also to the flux, giving Gauss's law:

$$\sum_{\text{closed surface}} E_{\perp} \Delta A = \frac{Q_{\text{encl}}}{\epsilon_0},$$

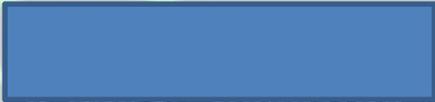
Statement Of Gauss's Law

The total electric flux through a closed surface is proportional to the enclosed charge

$$\oint \underline{E} \cdot \underline{dA} = \frac{\sum_{\text{inside}} q}{\epsilon_0}$$

Where:

\underline{E}	=	Electric Field
\underline{dA}	=	Area Vector
$\sum q$	=	Sum of all charges



Gauss' Law Summary

The electric field coming through a certain area is proportional to the charge enclosed.

$$\Phi_E = \int E dA = \frac{Q}{\epsilon_0}$$

Φ_E = Electric Flux (Field through an Area)

E = Electric Field

A = Area

Q = charge in object (inside Gaussian surface)

Electrostatic Potential

- The work done is given by:

$$\Delta W = F_x \Delta x$$

- The force F and the electric field E are oppositely directed, and we know that

$$F = -q \times E$$

- Therefore $E = -\Delta V / \Delta x$
- This is the ***potential gradient***.

Potential Due to a Point Charge

- Start with (set $V_f=0$ at ∞ and $V_i=V$ at R)

$$\Delta V = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} = -\int_i^f (E \cos 0^\circ) ds = -\int_R^\infty E dr$$

- We have

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

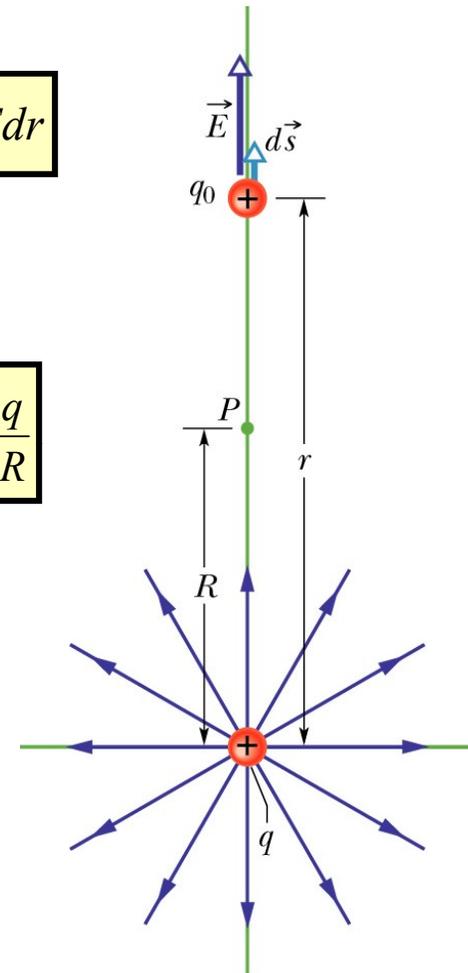
- Then

$$0 - V = -\frac{q}{4\pi\epsilon_0} \int_R^\infty \frac{1}{r^2} dr = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} \right]_R^\infty = -\frac{1}{4\pi\epsilon_0} \frac{q}{R}$$

- So

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

- A positively charged particle produces a positive electric potential.
- A negatively charged particle produces a negative electric potential



Potential due to a group of point charges

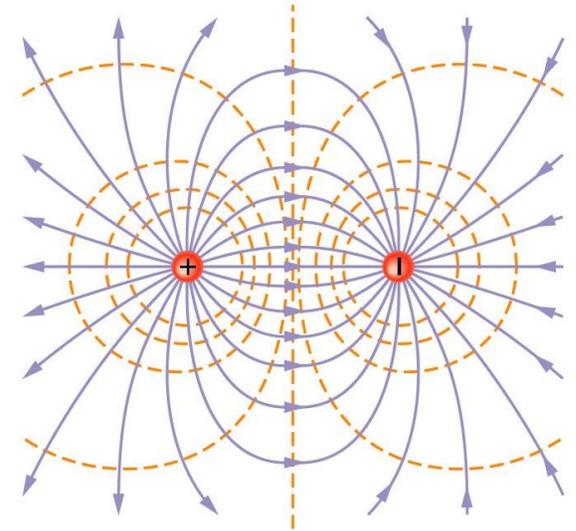
- Use superposition

$$V = -\int_{\infty}^r \vec{E} \cdot d\vec{s} = -\sum_{i=1}^n \int_{\infty}^r \vec{E}_i \cdot d\vec{s} = \sum_{i=1}^n V_i$$

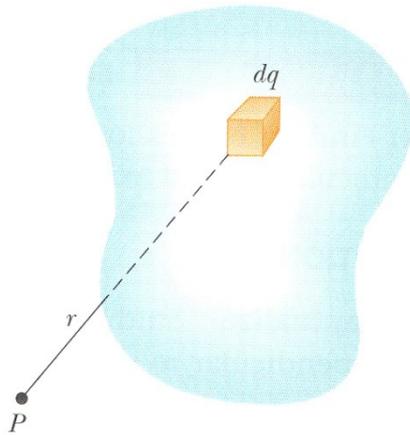
- For point charges

$$V = \sum_{i=1}^n V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}$$

- The sum is an algebraic sum, not a vector sum.



Potential due to a Continuous Charge Distribution



- Find an expression for dq :
 - $dq = \lambda dl$ for a line distribution
 - $dq = \sigma dA$ for a surface distribution
 - $dq = \rho dV$ for a volume distribution

- Represent field contributions at P due to point charges dq located in the distribution.

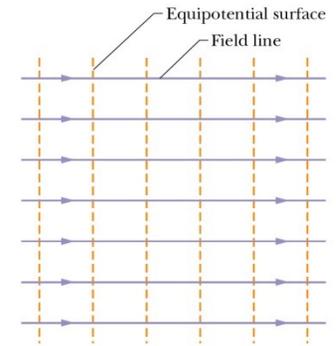
$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$

- Integrate the contributions over the whole distribution, varying the displacement as needed,

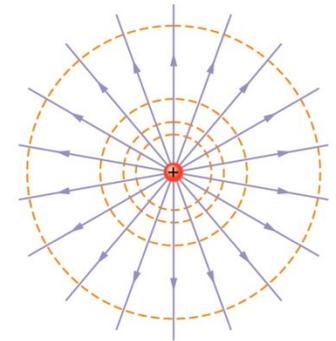
$$V = \int dV = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

Equipotential Surface

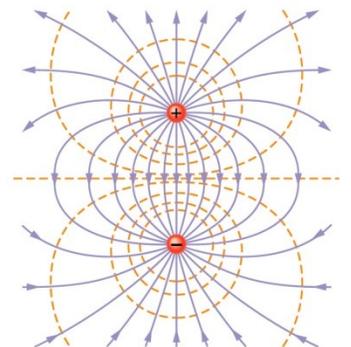
- The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.
- Equipotential surfaces are always perpendicular to electric field lines.
- No work is done by the electric field on a charged particle while moving the particle along an equipotential surface.



(a)



(b)



(c)

Thanks

